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Nonstationarity, nonlinear dependence, and prediction: An application to the Treasury Bill futures market

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NONSTATIONARITY, NONLINEAR DEPENDENCE, AND PREDICTION:
AN APPLICATION TO THE TREASURY BILL FUTURES MARKET

By

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Abstract of Dissertation Presented to the Graduate School of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

NONSTATIONARITY, NONLINEAR DEPENDENCE, AND PREDICTION: AN APPLICATION TO THE TREASURY BILL FUTURES MARKET

By

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Chairperson: Professor G.S. Maddala
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This study describes the time series properties of U.S. Treasury Bill futures prices with special emphasis on unit root nonstationarity, nonlinear dependence, and prediction. Although most research of financial markets assumes that market prices follow a specific martingale process, namely the random walk, recently researchers have begun to question this assumption. This assumption implies futures prices must contain a unit root, yet many studies are inconclusive or contradictory on this point. In chapter 2 several tests for nonstationarity are applied and it is shown that futures prices undoubtedly contain a unit root.

A more formal analysis of the random walk hypothesis is conducted in chapter 3 by looking at both linear and nonlinear dependence of first differences of prices. Nonparametric and parametric tests of linear dependence are conducted and the results indicate that the data contains no significant linear

dependence. However, when tests for nonlinear dependence were conducted, the results from every test indicated the nonlinear dependence.

Based on the results from chapter 3, chapter 4 estimates nonlinear models and uses them for prediction. In this chapter much is learned. First, some nonlinear models are excluded simply by their poor estimation performance. Second, when comparing the models' predictive performance to the random walk, it becomes clear that the nonlinearities of the data are exploitable. Two of four models are able to perform better than the random walk especially in shorter horizons. Third, the best nonlinear model is chosen after comparing the predictions of all the nonlinear models against each other. It is shown that the bilinear model is the best of the nonlinear models. Finally, it is shown that the bilinear model outperforms the popular autoregressive, conditional, heteroskedastic (ARCH) model.

CHAPTER 1
INTRODUCTION

General Background

The martingale process, i.e., a stochastic process in which the expected price in the next period equals the current price, has an established record in characterizing the random nature of futures prices. Samuelson (1965), assuming both perfect capital markets and an instantaneous adjustment property, was the first to formally develop a model where futures prices are characterized by a specific martingale process known as the random walk. Since then, many authors have tested the random walk property by testing first differences for serial independence.[1] The results, however, have been inconclusive. Rocca (1969) and Labys and Granger (1970) both concluded that the martingale process provides a good description of futures prices even though minor departures may be encountered. However, using both time and frequency domain tests, Cargill and Rausser (1972, 1975)

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[1] Note that for a time series of the variable x to be a martingale process the only requirement is that $E(x_{t+1})=x_t$ and $E(e_t)=0$ where $e_t=x_{t+1}-x_t$. But for x to be a random walk process the residual e_t must also have the property that $Cov(e_t,e_{t+k})=0$ for all k.

rejected the random walk hypothesis. Because previous time and frequency domain tests both assume that futures prices are normally distributed, Mann and Heifner (1976) used two nonparametric tests to test the random walk hypothesis. They also rejected the hypothesis after looking at prices for nine commodities over a twelve year span.

In addition to studies of the residuals of price differences, tests for nonstationarity can also be employed to address the same question. Recall that a martingale process is a stochastic process in which the expected price in the next period equals the current price. Then if futures prices can be described by this type of process, they should at least contain a unit root in their autoregressive representation. Goldenberg (1989) finds a unit root in daily S&P 500 futures prices. In addition, Doukas (1990) found that futures prices for some commodities, namely soybeans, soy meal, and soy oil, contain a unit root. These papers give some validity to the martingale hypothesis, but by themselves cannot be conclusive.

All of these studies of futures prices above have tried to investigate the martingale hypothesis or more specifically the random walk hypothesis by testing the existence of linear dependence. It is possible, however, that the random walk hypothesis may be violated by the existence of nonlinear dependence. Indirectly, some authors have addressed this possibility by showing that profitable trading rules may exist even when changes in futures prices are serially uncorrelated.

Leuthold (1972), in investigating the futures market for cattle, used filter rules to show that profitable trading rules existed for the period 1965-1970. In the same paper, he showed that these rules may exist even when spectral analysis indicates that price changes are random. Nonlinear dependence has been found in other financial data, but direct tests for nonlinear dependence in futures prices have not been conducted.

Because nonlinear dependence has been found in the residuals of price changes in other financial markets, it seems useful to test for nonlinear dependence in futures markets. There are several models that are good candidates for financial data, but one family of models, the autoregressive, conditional heteroskedastic (ARCH) family, has become the most popular univariate time series model. The cause of this popularity is unclear. Other models are just as easy to apply and have an intuitive appeal that is as good or better.

Purpose of the Study

Because the random walk hypothesis and time series properties of futures prices are still a subject of debate, the present dissertation examines the statistical nature of futures prices in detail. As the title of the dissertation suggests, nonstationarity, nonlinear dependence, and prediction will be the focus of the analysis. Given the size of the particular futures market chosen and the topics selected for discussion, the analysis will be sufficient to shed some light on the overall behavior of futures prices in financial markets.

In the second chapter, the property of nonstationarity is examined by first discussing and then applying tests for nonstationarity to the futures contracts chosen. Four different tests are used and some are used with different lag structures to account for any serial correlation found in the residuals of the tests' regression equations. Before concluding that the data are stationary or nonstationary, however, the appropriate Dickey-Fuller model of the data is considered. This entails analyzing which first order autoregressive representation, i.e., with no constant, just a constant, or a constant and a trend term, is the one that fits the data best. Test results indicate that first differences of prices are covariance stationary and give us a necessary condition to further investigate the random walk hypothesis.

Given the unanimous results from the tests for nonstationarity, the third chapter investigates the random walk hypothesis even further by applying parametric and nonparametric tests for linear dependence, a general test of dependence, and tests for nonlinear dependence to first differences of prices. As opposed to previous studies of futures prices, which used indirect tests for nonlinear dependence, direct tests for nonlinear dependence, which are based on new time series techniques, are used in this chapter. Among the tests used, I am first to apply a very powerful test known as the Brock, Dechert, and Scheinkman (BDS) test. This BDS test is based on the correlation integral which is used in physics as a measure of clustering. Significant linear dependence is rejected by all of the tests, but nonlinear dependence appears in every data set. This result leaves the random walk hypothesis in question and points to the use of nonlinear models as the most appropriate class of models to describe futures price data.

Modeling the data is taken up in the fourth chapter. Several nonlinear models are applied to the data, namely, an autoregressive, conditional heteroskedastic in mean (ARCH-M) model (Engle, Lilien, and Robins, 1987), a generalized autoregressive, conditional heteroskedastic in mean (GARCH-M) model (Bollerslev, 1986), a bilinear model (Granger and Andersen, 1978a), a time-varying parameter model, a time-series segmentation model (Sclove, 1983), and the stochastic, segmented trends model (Hamilton, 1989). First, the models are estimated. Because the time-varying parameter and time series segmentation models do not fit this data, they are discarded. The remaining models are estimated and used for out-of-sample prediction by reserving the last 50 days of data for each contract. Using two criteria, the mean square error of prediction and Theil's U statistic, the models' predictions are first compared to the prediction for a standard martingale, the random walk, and then compared to one another. The bilinear model predicts best and some conclusions are drawn as to the use of the family of ARCH models when modeling financial futures data.

Data Description

The data are daily settlement prices for 90-day U.S. Treasury Bill futures contracts. The contracts chosen for analysis in this dissertation were the five most recent contracts available at the start of my research. These contracts matured in third, sixth, ninth, and twelfth months of 1988 and the third month of 1989 and hereafter are referred to as contracts 88(3), 88(6), 88(9), 88(12), 89(3) respectively. After discarding the last month of trading for each contract to avoid dependencies caused by the convergence of futures prices to spot prices, there were approximately 450 observations for each contract.[2]

A simple reason that this particular futures market is chosen is that it is representative of all other financial futures markets, especially futures markets of other short-term credit instruments, by the dollar amount of transactions and volume traded on the market on any given day. In addition, it is the largest domestically traded futures

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[2] The first month of trading under contract 88(9) was characterized by dramatic upward and downward swings along with terribly low volumes of trading. For this reason this month of trading was also discarded to avoid any unexplainable dependencies that this behavior may cause to appear.

contract and is most often used as an indicator of future interest rates.

To investigate the random walk hypothesis, first differences are used in all of the tests, i.e., $e_t = P_t - P_{t-1}$, where P_t is the daily settlement price in time period t. To get a feel for the data, table 1.1 provides the summary statistics for these changes.

Table 1.1

SUMMARY STATISTICS FOR DAILY PRICE CHANGE
$e_t = P_t - P_{t-1}$

Contracts	88(3)	88(6)	88(9)	88(12)	89(3)
N	476	434	409	472	462
Mean	.0027	-.0010	-.0028	-.0007	-.0028
SD	.1107	.1121	.1073	.1038	.1033
Skewness	1.8789	.6724	1.5254	1.3058	1.3720
Kurtosis	9.9306	20.7811	19.8099	18.9414	20.9839
Maximum	1.06	1.01	.99	.97	.98
Minimum	-.36	-.77	-.55	-.60	-.64
T-stat	.0001	.0003	.00002	-.0002	-.0002

N indicates the number of observations and the T-statistic is from an OLS regression of daily price changes on time.

The skewness and kurtosis coefficients differ greatly from those found on a normal distribution (0 and 3 respectively). In all of the samples, the density is skewed to the right and the size of the kurtosis coefficients indicates that the density is far more peaked around its center than the density of a normal random variable (leptokurtic). Note that if the density of price changes is nonnormal, then satisfying simple tests for randomness will only indicate that e_t is uncorrelated over time. Without normality, statistical independence cannot be inferred from these results.

In figures 1.1 through 1.5, the levels of futures prices for contracts 88(3) through 89(3) are graphed against time and in figures 1.6 through 1.10 price changes are graphed against time and presented in the same order. The levels of the data appear to be autocorrelated both negatively and positively over different periods of time. In addition, there seem to be long periods where futures prices move in one direction. Hence, the statistical models proposed by Sclove (1983) and/or Hamilton (1989) , which will be discussed in chapter 4, seem to be applicable.

The changes in futures prices, on the surface, are less informative, although it appears that the data are bounded and linearly independent. In addition, a simple inspection of the way the amplitude changes over time may lead one to believe that the data could have been generated by some linear martingale process. However, it is also known that graphs of bilinear, ARCH, or GARCH processes, processes that will also be discussed in chapter 4, could look this way. Because it is difficult to visually detect whether the amplitude of the series changes over time or is related over time, we will leave it up to the estimation of models in chapter 4 for more information. The ability of a model to predict as well as some standard residual diagnostic tests should distinguish the most appropriate model for the data.

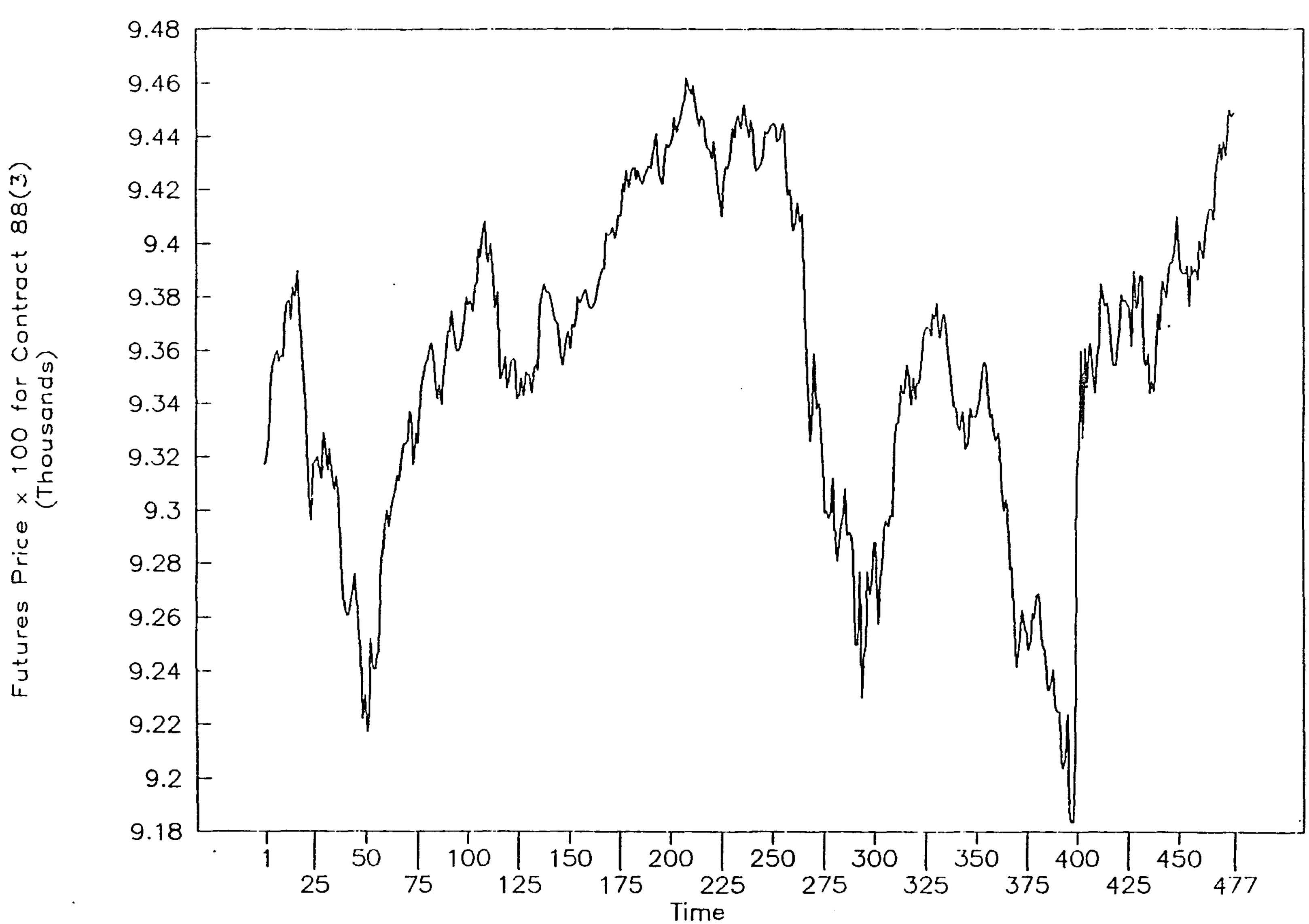

FIGURE 1.1 Futures Prices For Contract 88(3)

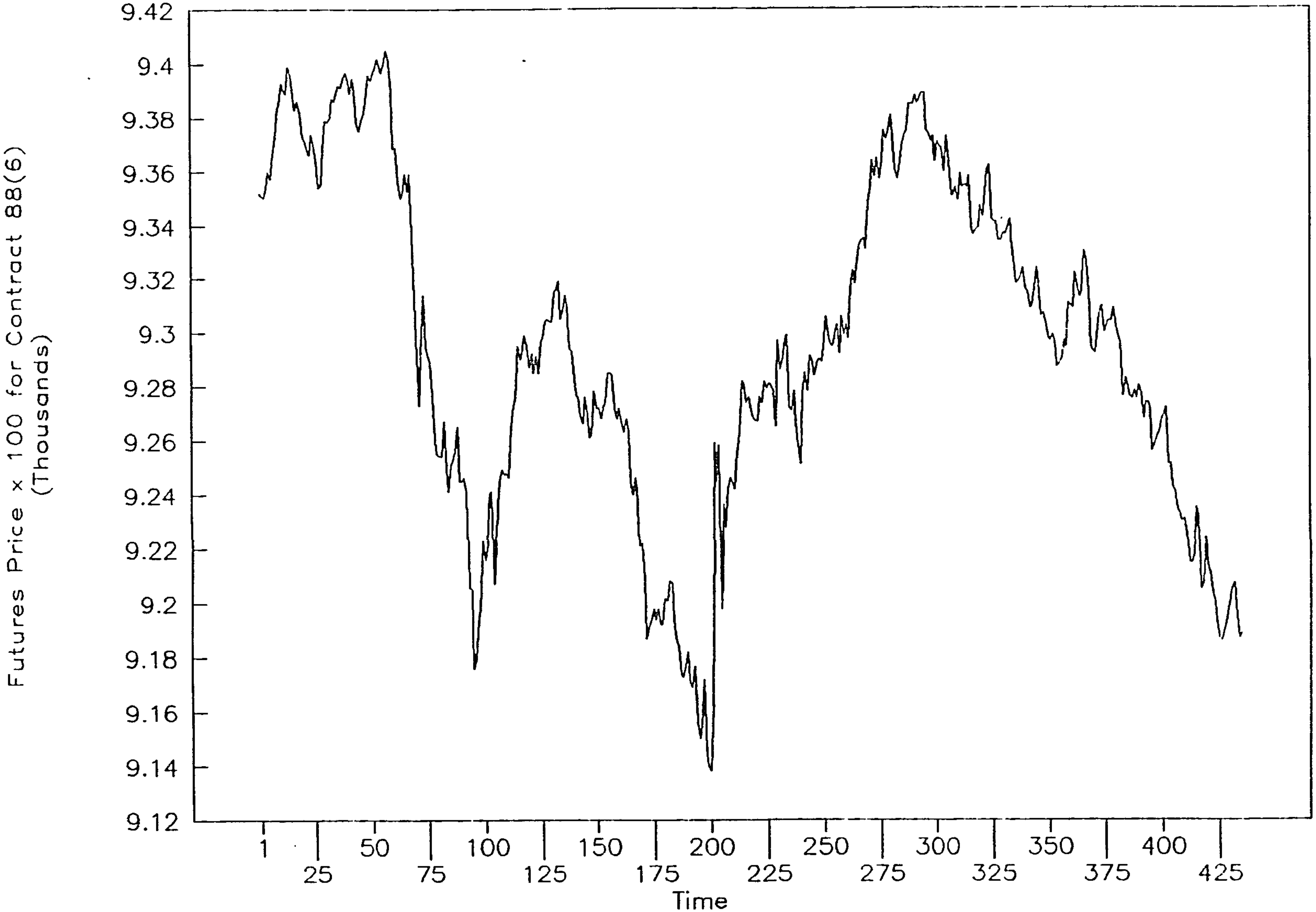

FIGURE 1.2 Futures Prices For Contract 88(6)

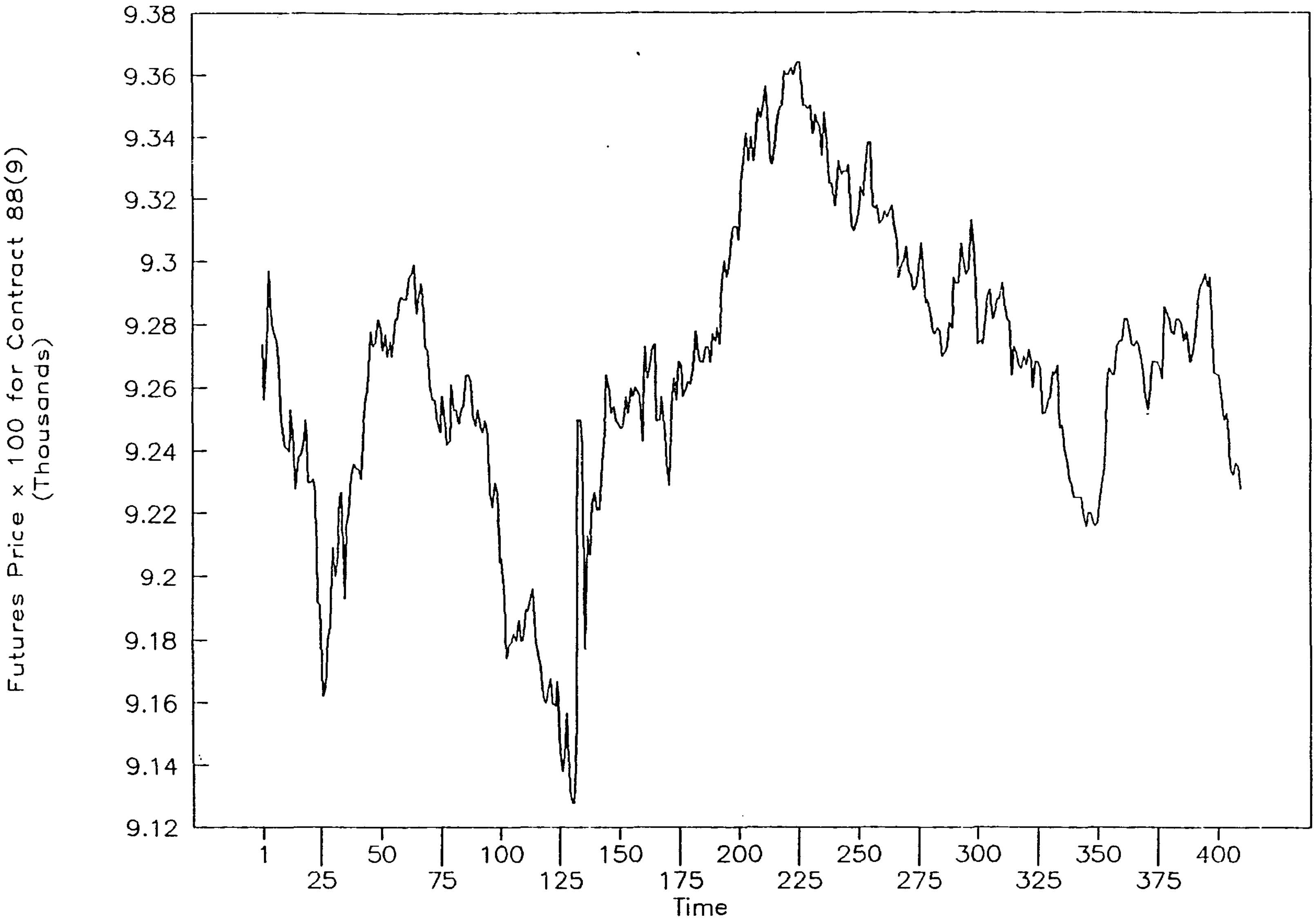

FIGURE 1.3 Futures Prices For Contract 88(9)

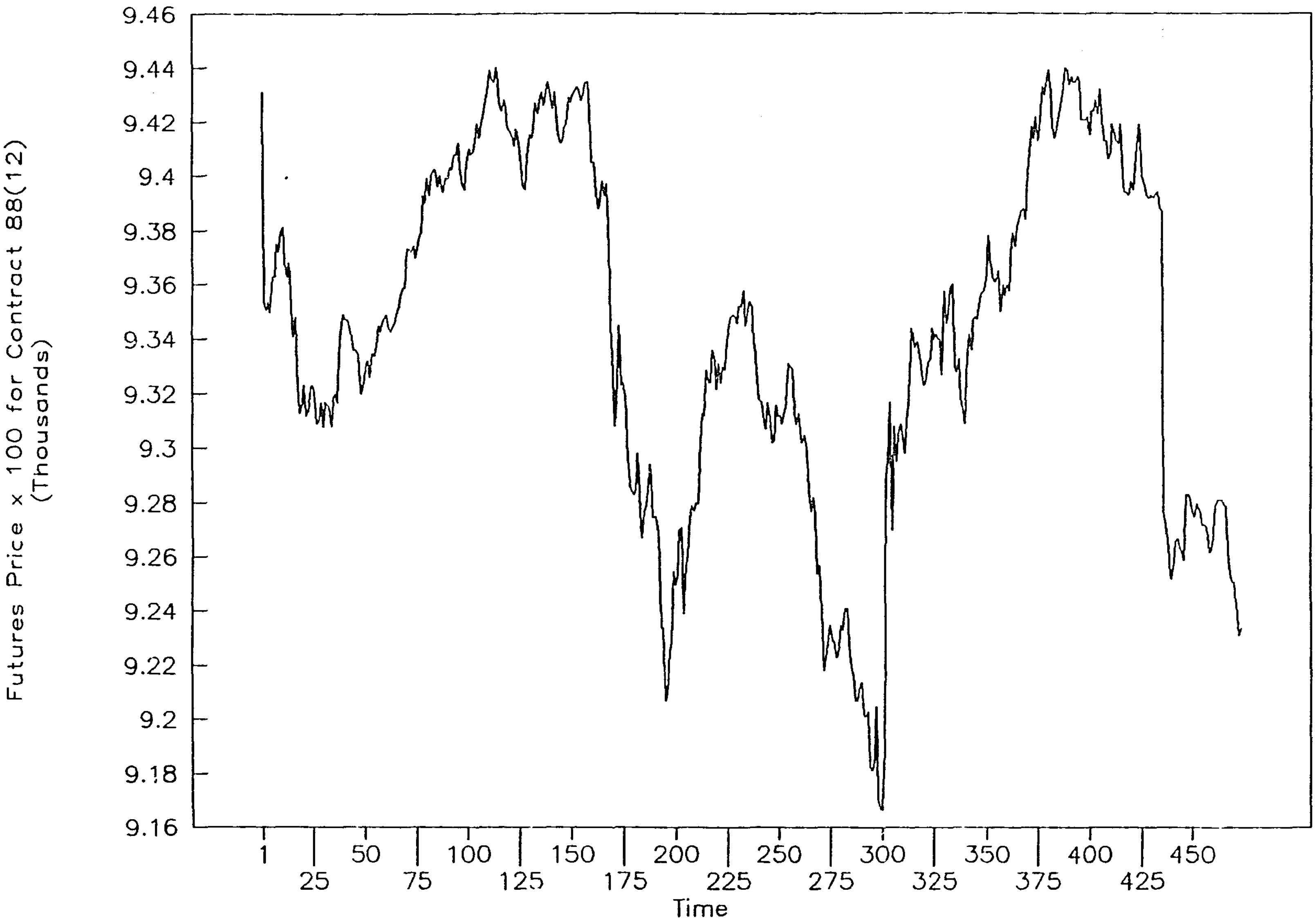

FIGURE 1.4 Futures Prices For Contract 88(12)

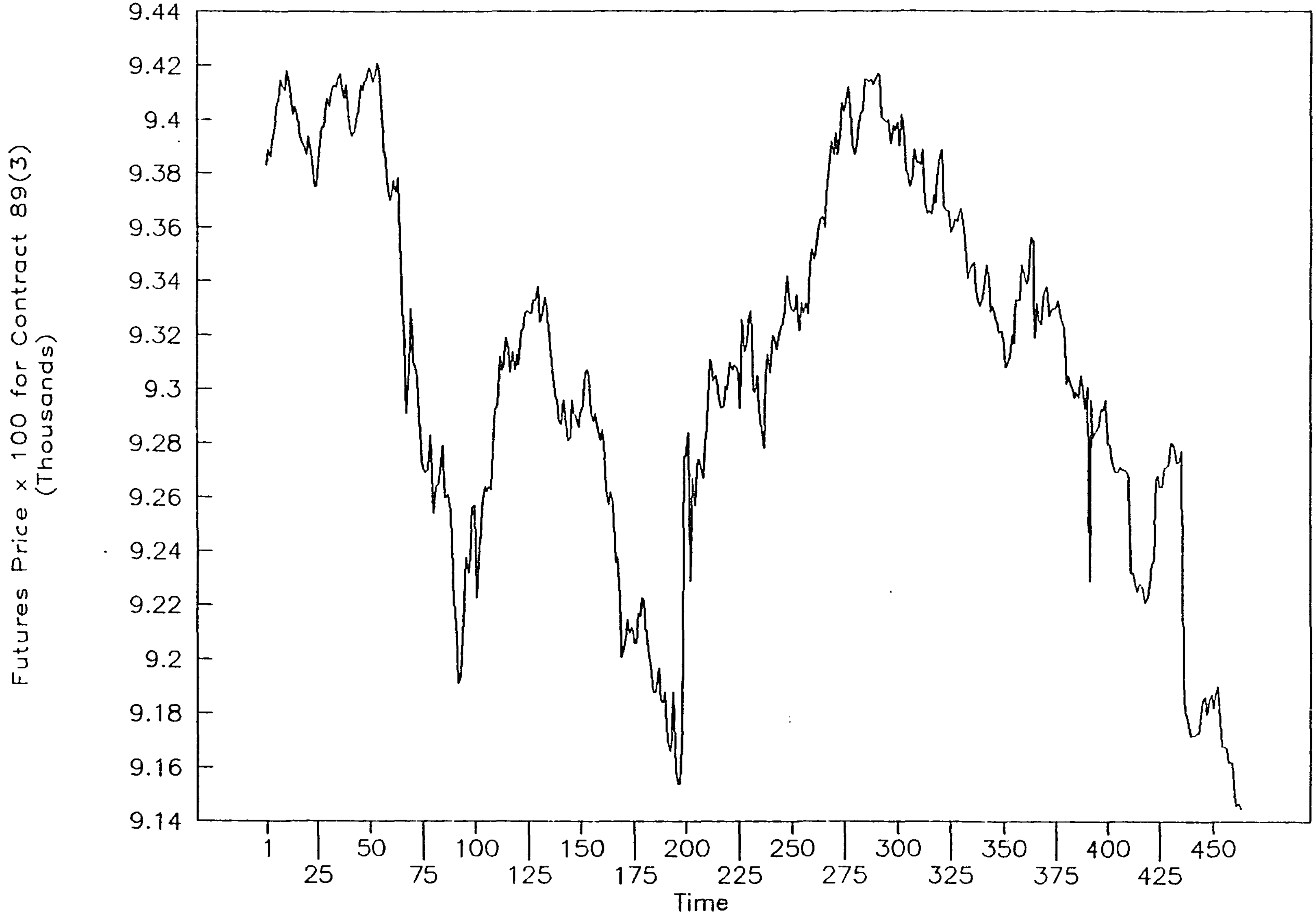

FIGURE 1.5 Futures Prices For Contract 89(3)

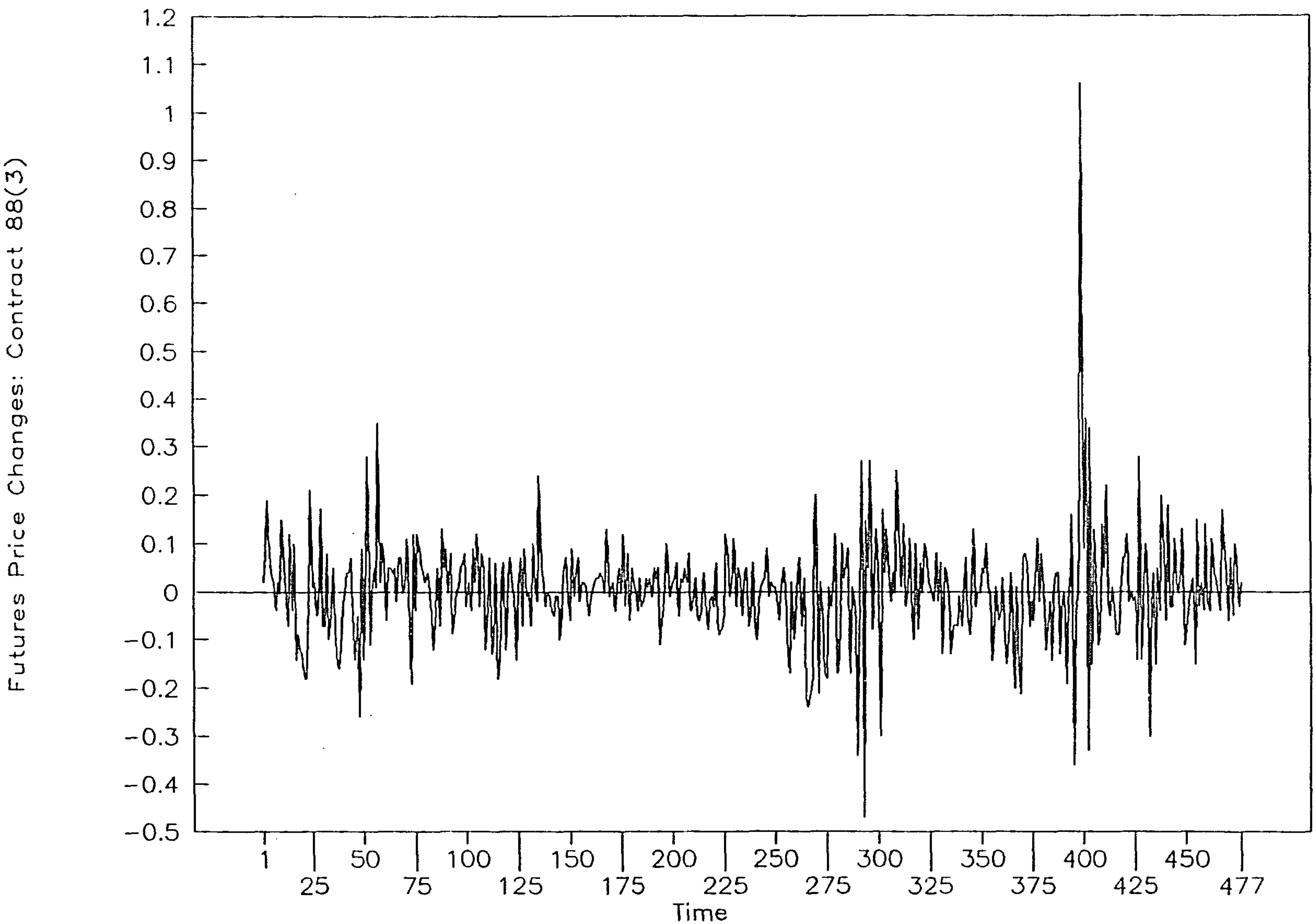

FIGURE 1.6 Changes In Futures Prices For Contract 88(3)

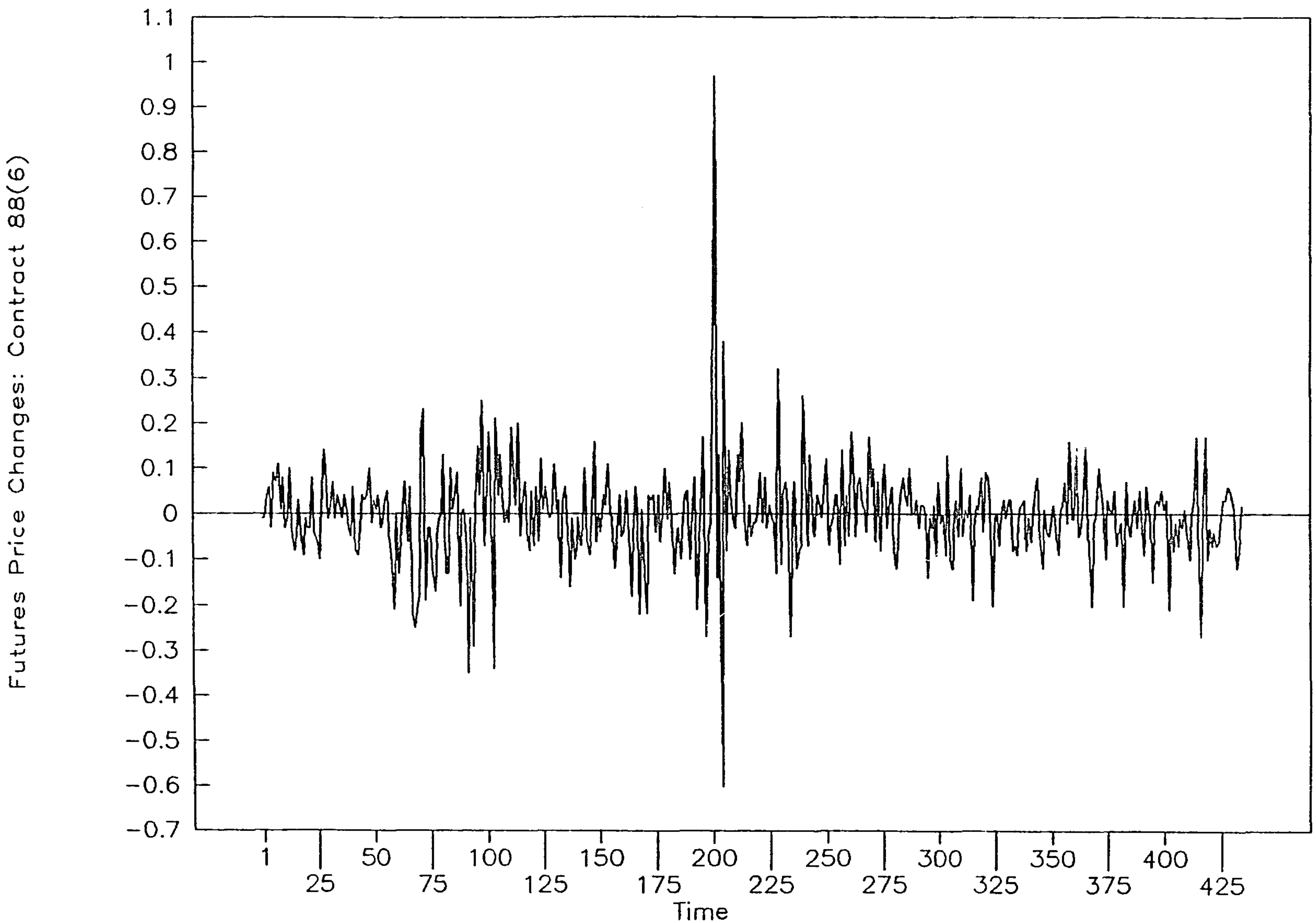

FIGURE 1.7 Changes In Futures Prices For Contract 88(6)

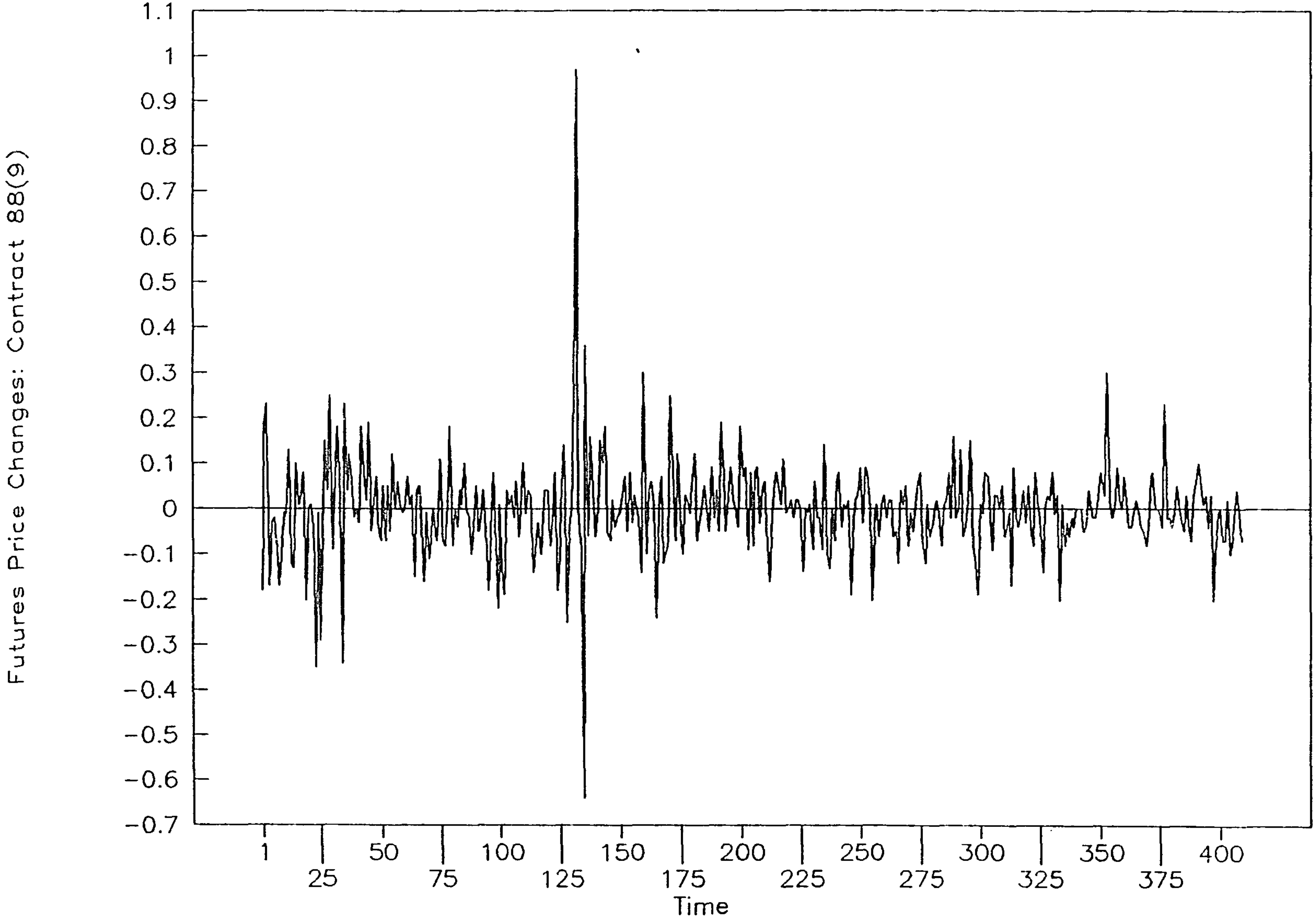

FIGURE 1.8 Changes In Futures Prices For Contract 88(9)

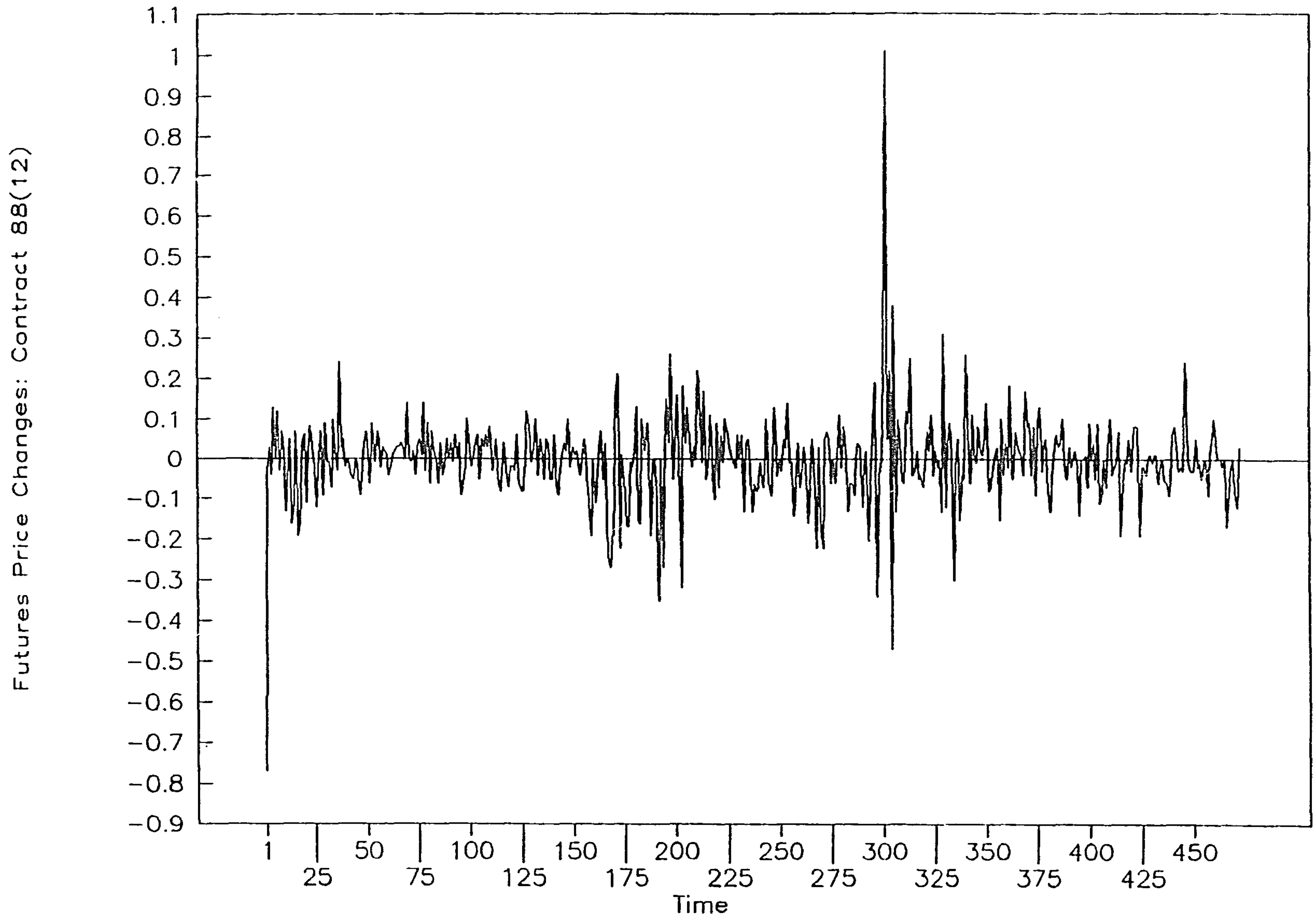

FIGURE 1.9 Changes In Futures Prices For Contract 88(12)

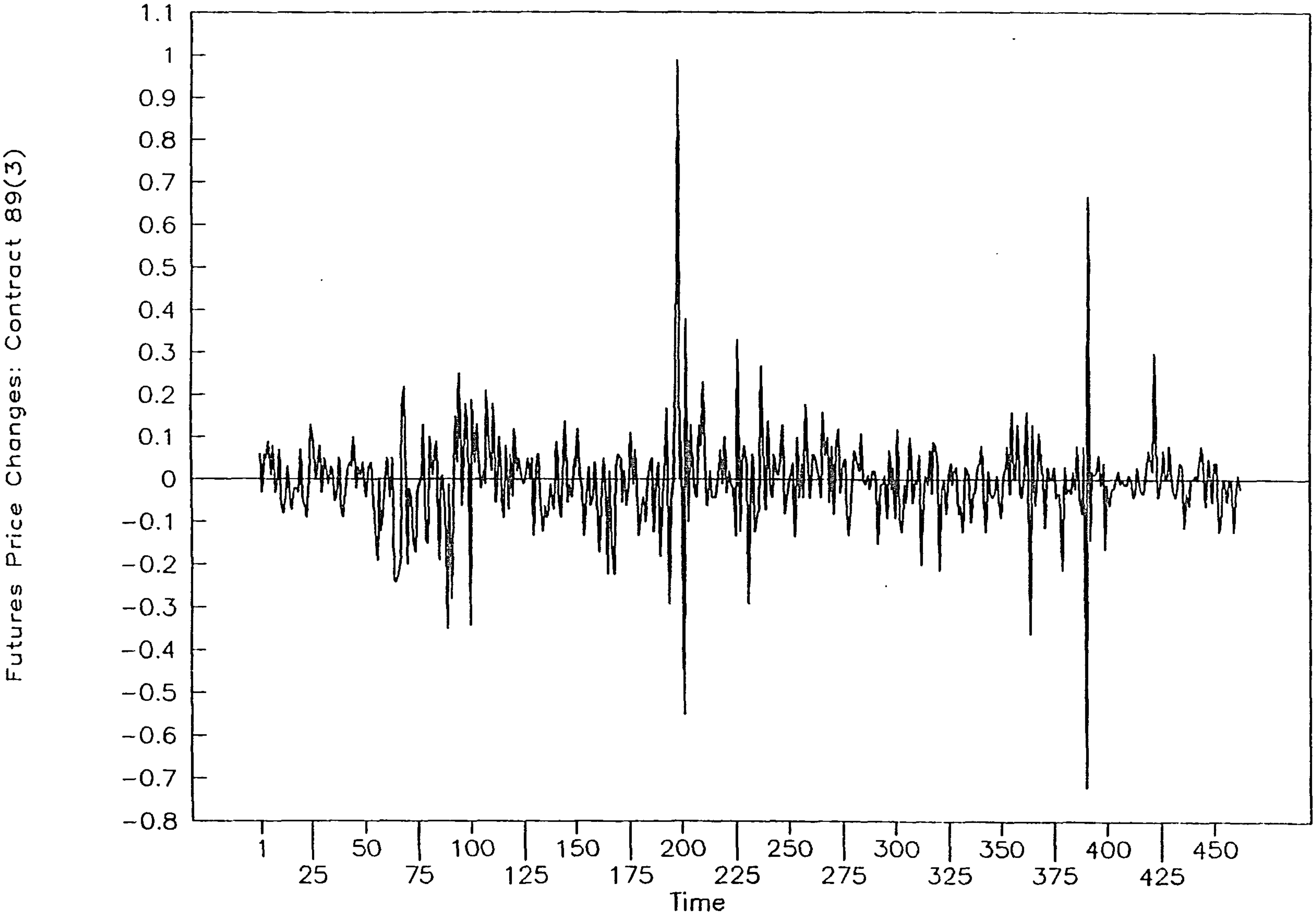

FIGURE 1.10 Changes In Futures Prices For Contract 89(3)

Layout of the Dissertation

As the title of this thesis suggests, U.S. Treasury Bill futures data are the primary focus of the analysis. Given the data, the thesis has three objectives; characterization, estimation, and prediction. First, the time series properties of data are characterized. By this it is meant that the properties of nonstationarity and nonlinearity are investigated. There has been somewhat of an ongoing curiosity as to whether futures prices contain a unit root.[3] The property of nonstationarity is addressed and several different tests for unit roots are applied to the data in the second chapter.

In the third chapter, the property of nonlinear dependence is examined. In a preliminary analysis of some summary statistics of the data, it seems likely that nonlinear dependence is an intrinsic part of the data. Since this property has recently been found in other financial markets, namely, the foreign exchange rate market and stock market, it raises even more suspicion. First, the data are checked for any dependence, linear or nonlinear, by applying some general tests of dependence. Then the data are purged of any linear dependence by regressing price differences on ten lags using ordinary least squares estimation. This is done to avoid any sensitivity that tests for nonlinearity may have for linear

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[3] A full discussion of this curiosity is given in the introduction to chapter 2.

dependence. The residuals from this procedure are then examined for nonlinear dependence.

It is hypothesized that the data have at least one of two types of nonlinear dependence, multiplicative and/or additive. Using a new test, known as the third order moment test, the nonlinear dependence found in the data is then classified as one of these types. Whichever type of dependence is found, this information can then be used to identify the most appropriate nonlinear models.

In the fourth chapter, the second objective, estimation, is addressed. Here, I estimate several univariate time series models, evaluate their suitability, and consider the way nonlinearities enter the data and their implications for the importance of nonlinearities.

Prediction using the nonlinear models is the last objective and is also encountered in the fourth chapter. It is here that two important questions are answered. First, can the nonlinearities that exist in the data be exploited to earn profits?[4] Securities traders as well as other researchers, who have not been able to use nonlinear dependencies to assist in predicting the mean of the process in other financial markets, will find both this question and its answer interesting. This question is answered by comparing predictions from the nonlinear models to the prediction from

[4] Note that the ability to make profits will depend not only on a successful model, but on the costs of trading securities.

a simple linear model since it is believed that futures prices behave like random walks. The second question is concerned with the most appropriate nonlinear model. One particular family of nonlinear models, called ARCH models, have been used and sometimes abused by researchers when modeling financial data. It is in this chapter that ARCH models are compared with other nonlinear models of futures prices.

In the conclusion of this dissertation, several important discoveries are pointed to. The results, taken as a whole, should prove useful for researchers of futures markets and will offer food for more research on the time series properties of futures markets in general.

CHAPTER 2
NONSTATIONARITY

Introduction

It has long been assumed that changes in futures prices are covariance stationary processes (see, for example, Telser, 1967; Stevenson and Bear, 1970; Martell and Helms, 1978; and Trevino and Martell, 1984). By this it is meant that, although the probability distribution of the series may change over time, the mean and variance of the series do not change with time and the covariance between two realizations in time depends only on the time difference, not on the time instant. This assumption of covariance or wide-sense stationarity is necessary for time-invariant representations of futures prices in terms of their conditional expectations. In addition, for any of the ergodic theorems to hold, stationarity is necessary. Cargill and Rausser (1975), Stevenson and Bear (1970), and Alexander (1961) report trends in commodity futures prices, Goldenberg (1989) finds a unit root in daily S&P 500 futures prices, and Doukas (1990) finds a unit root in daily soy meal, soybean, and soy oil futures prices. The conflicting discoveries on the issue of stationarity in futures prices suggest that a formal test of the data is required before any other time series analyses may be conducted. In this chapter I set out to establish whether or not futures prices are nonstationary.

For the simplest of unit root tests, under the null hypothesis, the assumption is that the data follow a random walk, i.e., $P_t = P_{t-1} + e_t$, where P_t is the daily settlement price of a futures contract in period t and e_t is an independently and identically distributed (i.i.d.) normal random variable with mean 0 and variance σ^2. A conventional and easily applied test for nonstationarity is the DF test, suggested by Dickey and Fuller (1979). This test, however, is somewhat limited since the error term is assumed to be strictly i.i.d. $N(0,\sigma^2)$ under the null hypothesis. Recently, a lot of effort has been exerted on developing tests that relax this assumption. The Dickey-Fuller test for unit roots in the standard AR(1) model can be generalized to test for unit roots in an AR(p) model. The Augmented Dickey-Fuller (ADF) test, suggested by Said and Dickey (1984), extends the Dickey-Fuller test to account for serial correlation that is typically produced by autoregressive moving average (ARMA) models. Two tests that nonparametrically adjust the DF test to correct for infinite-dimensional nuisance parameters associated with e_t are the Z_α and Z_t tests suggested by Phillips (1987) and Phillips and Perron (1988) respectively. The Phillips' tests are designed to handle generalized forms of serial correlation and/or heteroskedasticity that may be contained in e_t.

In general, unit root tests have received a lot of attention in contemporary research and have become major tools in time series analyses. This chapter proceeds as follows. In the next section, the details of the unit root tests applied in this chapter are given. In the section entitled "Testing the Data for Nonstationarity," I present the results from applying these tests to the data. I distinguish the appropriate Dickey-Fuller model to be used in the unit root tests, the most suitable random walk model to be used in subsequent chapters, and briefly conclude in the last section.

Tests for Nonstationarity

A major branch of the literature contains tests that are all based on the following observation. Consider the simplest data generation process that allows one to discuss the concept behind these tests:

$$P_t = \rho P_{t-1} + u_t; \quad u_t \sim i.i.d.(0, \sigma_u^2) \qquad \textbf{(2.1)}$$

$$P_0 = 0.$$

If the null hypothesis is H_0: $\rho = \rho_0$, where $|\rho_0| < 1$, then the t-statistic is asymptotically normally distributed. If $\rho_0 = 1$, then the test statistic is no longer asymptotically normal. The resulting distribution is not even symmetric.

Critical values for the hypothesis of a unit root are found by using Monte Carlo simulation. They were first tabulated by Dickey and presented in Fuller (1976). This simple test is known as the Dickey-Fuller (DF) test. The critical values that they tabulated, which are presented below, are based on the following three models.

$$\Delta P_t = (\rho-1)P_{t-1} + u_t \tag{2.2}$$

$$\Delta P_t = \alpha + (\rho-1)P_{t-1} + u_t \tag{2.3}$$

$$\Delta P_t = \alpha + \gamma t + (\rho-1)P_{t-1} + u_t \tag{2.4}$$

where $\Delta P_t = P_t - P_{t-1}$. If we let the sample size = T and a = the cumulative probability, then the critical values, in tables 2.1-2.3 below, correspond to the statistic $(\hat{\rho}-1)/SE(\hat{\rho})$ for models 2.2-2.4 respectively.

Table 2.1[1]

Empirical Cumulative Distribution of $(\hat{\rho}-1)/SE(\hat{\rho})$ for Model 2.2

T	a= 0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-2.66	-2.26	-1.95	-1.60	.92	1.33	1.70	2.16
50	-2.62	-2.25	-1.95	-1.61	.91	1.31	1.66	2.08
100	-2.60	-2.24	-1.95	-1.61	.90	1.29	1.64	2.03
250	-2.58	-2.23	-1.95	-1.62	.89	1.29	1.63	2.01
500	-2.58	-2.23	-1.95	-1.62	.89	1.28	1.62	2.00
∞	-2.58	-2.23	-1.95	-1.62	.89	1.28	1.62	2.00

[1] Tables 2.1-2.6 are found in Fuller (1976).

Table 2.2

Empirical Cumulative Distribution of $(\hat{\rho}-1)/SE(\hat{\rho})$ for Model 2.3

T	a= 0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-3.75	-3.33	-3.00	-2.63	-0.37	0.00	0.34	0.72
50	-3.58	-3.22	-2.93	-2.60	-0.40	-0.03	0.29	0.66
100	-3.51	-3.17	-2.89	-2.58	-0.42	-0.05	0.26	0.63
250	-3.46	-3.14	-2.88	-2.57	-0.42	-0.06	0.24	0.62
500	-3.44	-3.13	-2.87	-2.57	-0.43	-0.07	0.24	0.61
∞	-3.43	-3.12	-2.86	-2.57	-0.44	-0.07	0.23	0.60

Table 2.3

Empirical Cumulative Distribution of $(\hat{\rho}-1)/SE(\hat{\rho})$ for Model 2.4

T	a= 0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-4.38	-3.95	-3.60	-3.24	-1.14	-0.80	0.50	0.15
50	-4.15	-3.80	-3.50	-3.18	-1.19	-0.87	0.58	0.24
100	-4.04	-3.73	-3.45	-3.15	-1.22	-0.90	0.62	0.28
250	-3.99	-3.69	-3.43	-3.13	-1.23	-0.92	0.64	0.31
500	-3.98	-3.68	-3.42	-3.13	-1.24	-0.93	0.65	0.32
∞	-3.96	-3.66	-3.41	-3.12	-1.25	-0.94	0.66	0.33

Concurrently, Dickey and Fuller presented an expanded version of the DF test in Dickey and Fuller (1979). The initial test discussed above handles the AR(1) case, whereas the expanded version handles the AR(p) case. For P_t as an AR(p) process

$$P_t = \sum_{j=1}^{p} \rho_j P_{t-j} + \epsilon_t \tag{2.5}$$

a test can be constructed by using the regression model

$$\Delta P_t = (\rho-1) P_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta P_{t-j} + u_t \tag{2.6}$$

where $\Delta P_t = P_t - P_{t-1}$. The test statistic, as in the DF test, is the t-statistic for the coefficient of P_{t-1}. This model can also be extended to include a constant and a trend just as the models in 2.3 and 2.4. The model presented in (2.6) is known as the Augmented Dickey-Fuller (ADF) test.

Said and Dickey (1984) show that the ADF test can also be used to test for unit roots even when the error term u_t follows an MA process or a general ARMA(p,q) process, so long as the ARMA process is stationary and invertible. The only proviso is that p rises with the sample size T so that there exists numbers c>0 and r>0, such that $cp > T^{1/r}$ and $T^{-1/3}p \rightarrow 0$. Theoretically, many choices of p can satisfy this requirement. Schwert (1989) showed that tests for nonstationarity are affected by the presence of a moving average or invertible autoregressive parameter in the residuals of Dickey-Fuller models given in equations (2.2a-c). However, depending on the value of this parameter, different lengths of p are appropriate. Hence, Schwert (1989) suggests that p be chosen according to the following two equations, one which gives a shorter length of p and the other a longer length.

$$l_4 = INT[4(T/100)^{1/4}] \tag{2.7}$$

$$l_{12} = INT[12(T/100)^{1/4}] \tag{2.8}$$

where INT[.] denotes the integer component. By using these two values for "l", two ADF statistics, ADF[4] and ADF[12], corresponding to l_4 and l_{12} respectively, are constructed.

The adjustment to the DF test that the ADF test makes is simply one to retain the validity of the assumption of white noise errors in the DF regression. The Z_α and Z_t tests, suggested by Phillips (1987) and Phillips and Perron (1988) respectively, are tests that, instead of adjusting the DF regression before estimation, modify the DF regression after estimation through a nonparametric adjustment. Hence, the error term is not assumed to follow a white noise process. Like the ADF test, these tests handle possible autocorrelation that may exist between the first differences of P_t. In addition, these tests make allowances for heteroskedasticity that the residuals of the DF regression may exhibit. Formally, to find both the Z_α and Z_t test statistics one starts from the DF regression, i.e.,

$$\Delta P_t = \alpha + \beta P_{t-1} + \epsilon_t \tag{2.9}$$

The Z_α statistic is equal to $T\hat{\beta}$ - AD_α where $\hat{\beta}$ is the OLS estimate of β in equation (2.9) and AD_α is defined as

$$AD_\alpha = \frac{\frac{1}{2}(s_{Tl}^2 - s_\epsilon^2)}{\frac{1}{T}\sum_{t=2}^{T}(P_{t-1} - \overline{P}_{-1})^2} \tag{2.10}$$

where s_ϵ^2 is the maximum likelihood estimate of the sample variance of the residuals ϵ_t. That is

$$s_{\epsilon}^{2} = \frac{1}{T}\sum_{t=1}^{T} \epsilon_{t}^{2} \tag{2.11}$$

and

$$s_{Tl}^{2} = \frac{1}{T}\sum_{t=1}^{T} \epsilon_{t}^{2} + \frac{2}{T}\sum_{j=1}^{l} \omega_{jl} \sum_{t=j+1}^{T} \epsilon_{t}\epsilon_{t-j}. \tag{2.12}$$

Newey and West (1987) suggested the weights $\hat{\omega}_{jl} = \{1 - j/(l + 1)\}$ to ensure that the estimate of the variance s_{Tl}^2 is positive as well as consistent. The condition on the lag structure is simply that $l \to \infty$ as $T \to \infty$, such that l is $o(T^{1/4})$. Note that Schwert's (1989) suggestion satisfies this condition. Hence, I use, as Schwert did, l_4 and l_{12} to calculate the appropriate number of lags in s_{Tl}^2 and report two Z_α statistics. To calculate the statistic, ϵ_t is replaced by its OLS estimate $\hat{\epsilon}_t$. The Z_α test uses critical values that are used for the alternative expression for the DF test statistic, $T\hat{\beta}$. These critical values are given in tables 2.4-2.6 below.

The Z_t statistic is defined as

$$Z_t = t_\beta\left(\frac{s_\epsilon}{s_{Tl}}\right) - \frac{\frac{1}{2}(s_{Tl}^2 - s_\epsilon^2)}{\sqrt{\frac{s_{Tl}^2}{T^2}\sum_{t=2}^{T}(P_{t-1} - \bar{P}_{-1})^2}} \tag{2.13}$$

where t_β is the Student t-statistic of $\hat{\beta}$ from the simple DF regression and s_ϵ^2 and s_{Tl}^2 are defined as above. As for the Z_α, there are two statistics for the Z_t statistic, one corresponding to each lag structure, l_4 and l_{12}. The critical values for the Z_t statistic are identical to those used for the DF and ADF tests above.

Table 2.4

Empirical Cumulative Distribution of $T\hat{\beta}$ for Model 2.2

T	a= 0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-11.9	-9.3	-7.3	-5.3	1.01	1.40	1.79	2.28
50	-12.9	-9.9	-7.7	-5.5	0.97	1.35	1.70	2.16
100	-13.3	-10.2	-7.9	-5.6	0.95	1.31	1.65	2.09
250	-13.6	-10.3	-8.0	-5.7	0.93	1.28	1.62	2.04
500	-13.7	-10.4	-8.0	-5.7	0.93	1.28	1.61	2.04
∞	-13.8	-10.5	-8.1	-5.7	0.93	1.28	1.60	2.03

Table 2.5

Empirical Cumulative Distribution of $T\hat{\beta}$ for Model 2.3

T	a= 0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-17.9	-14.6	-12.5	-10.2	-0.76	0.01	0.69	1.40
50	-18.9	-15.7	-13.3	-10.7	-0.81	-0.07	0.53	1.22
100	-19.8	-16.3	-13.7	-11.0	-0.83	-0.10	0.47	1.14
250	-20.3	-16.6	-14.0	-11.2	-0.84	-0.12	0.43	1.09
500	-20.5	-16.8	-14.0	-11.2	-0.84	-0.13	0.42	1.06
∞	-20.7	-16.9	-14.1	-11.3	-0.85	-0.13	0.41	1.04

Table 2.6

Empirical Cumulative Distribution of $T\hat{\beta}$ for Model 2.4

T	a= 0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-22.9	-19.3	-17.9	-15.6	-3.66	-2.51	-1.53	-0.43
50	-25.7	-22.4	-19.8	-16.8	-3.71	-2.60	-1.66	-0.65
100	-27.4	-23.6	-20.7	-17.5	-3.74	-2.62	-1.73	-0.75
250	-28.4	-24.4	-21.3	-18.0	-3.75	-2.64	-1.78	-0.82
500	-28.9	-24.8	-21.5	-18.1	-3.76	-2.65	-1.78	-0.84
∞	-29.5	-25.1	-21.8	-18.3	-3.77	-2.66	-1.79	-0.87

Both Z_α and Z_t^* test statistics, as the DF and ADF tests, can be based upon the three different models, 2.2-2.4, above. However, model 2.4 must be modified when used for formulating the Z statistics. We modify 2.4 as follows:

$$\Delta P_t = \alpha + \gamma(t-T/2) + (\rho-1)P_{t-1} + \epsilon_t. \qquad \textbf{(2.4')}$$

<u>Testing the Data for Nonstationarity</u>

In this section I apply the tests described above to the Treasury Bill futures data. The three regression models listed in 2.2-2.4, using 2.4' where appropriate, are employed under each test. In addition, the lag structures l_4 and l_{12} are used for the ADF, Z_α, and Z_t tests. In total, twenty-one statistics will be presented, seven for each model 2.2-2.4. Tables 2.7-2.9 correspond to the models 2.2-2.4 respectively.

<u>Table 2.7</u>

Unit Root Tests with Zero Mean and Trend under the Ho
(Model 2.2)

Tests	88(3)	88(6)	88(9)	88(12)	89(3)
DF	0.538	-0.202	-0.492	-0.535	-0.591
ADF(4)	0.335	0.087	-0.689	-0.604	-0.730
ADF(12)	0.276	0.223	-0.485	-0.668	-0.458
Zα(4)	-1.278	-0.155	1.604	0.883	1.916
Zα(12)	0.065	0.019	-0.026	-0.011	-0.037
Zt(4)	0.160	-0.238	-0.153	-0.361	-0.247
Zt(12)	-0.139	-0.467	-0.355	-0.554	-0.299

88(3)-89(3) denotes the five contracts. All test statistics in the table, except those for the $Z\alpha$ test, should be compared to the critical values found in table 2.1. The critical values for the $Z\alpha$ test should be compared to the critical values found in table 2.4. * and ** denotes a rejection of the Ho for a one-sided test at the 5% and 1% levels of significance respectively. Note that large positive statistics indicate a rejection of a unit root but not of nonstationarity.

Table 2.8

Unit Root Tests with Nonzero Mean under the Ho
(Model 2.3)

Tests	88(3)	88(6)	88(9)	88(12)	89(3)
DF	-1.744	-2.024	-1.948	-1.925	-1.647
ADF(4)	-1.913	-1.686	-1.766	-1.783	-1.181
ADF(12)	-2.059	-2.149	-2.214	-2.398	-1.518
Zα(4)	-8.486	-7.906	-6.177	-6.855	-5.763
Zα(12)	-6.846	-7.412	-7.468	-8.370	-7.238
Zt(4)	-1.813	-2.016	-1.993	-1.931	-1.582
Zt(12)	-1.937	-2.044	-1.947	-1.938	-1.633

See notes under table 2.7. The critical values are found in tables 2.2 and 2.5.

Table 2.9

Unit Root Tests with Nonzero Mean and Trend under the Ho
(Model 2.4)

Tests	88(3)	88(6)	88(9)	88(12)	89(3)
DF	-1.745	-2.027	-1.945	-1.977	-1.679
ADF(4)	-1.911	-1.684	-1.756	-1.816	-1.194
ADF(12)	-2.044	-2.146	-2.214	-2.393	-1.486
Zα(4)	-8.497	-7.893	-6.190	-7.268	-5.863
Zα(12)	-6.874	-7.425	-7.479	-7.848	-7.389
Zt(4)	-1.816	-2.019	-1.991	-1.987	-1.618
Zt(12)	-1.932	-2.042	-1.945	-1.987	-1.658

See notes under table 2.7. The critical values are found in tables 2.3 and 2.6.

The test statistics applied to every model unanimously indicate that the data is nonstationary and the unit root hypothesis cannot be rejected. To be sure, however, I conduct the same unit root tests on the first differences of prices. These results are given in tables 2.10-2.13. Regardless of the data set, the test used, or the model specified, at the 5% level of significance the tests rejects the null hypothesis that first differences of futures prices contain a unit root. Hence, one can be more certain that the levels of futures prices are integrated of order 1 or that they contain a unit root.

Table 2.10

Unit Root Tests with Zero Mean and Trend under the Ho Using First Differences of Prices (Model 2.2)

Tests	88(3)*	88(6)	88(9)	88(12)	89(3)
DF	-21.44	-21.91	-22.79	-22.44	-21.56
ADF(4)	-8.56	-8.46	-8.38	-8.62	-8.74
ADF(12)	-4.11	-4.25	-3.96	-3.81	-4.21
Zα(4)	-499.7	-435.0	-448.9	-472.2	-420.3
Zα(12)	-536.9	-469.8	-490.8	-511.3	-426.5
Zt(4)	-20.27	-21.66	-23.37	-23.25	-24.18
Zt(12)	-19.44	-20.11	-21.19	-21.49	-23.83

88(3)-89(3) denotes the five contracts. All test statistics in the table, except those for the Zα test, should be compared to the critical values found in table 2.1. The critical values for the Zα test should be compared to the critical values found in table 2.4.

Table 2.11

Unit Root Tests with Nonzero Mean under the Ho Using First Differences of Prices (Model 2.3)

Tests	88(3)	88(6)	88(9)	88(12)	89(3)
DF	-21.43	-21.88	-22.78	-22.43	-21.54
ADF(4)	-8.56	-8.45	-8.40	-8.64	-8.76
ADF(12)	-4.09	-4.26	-3.98	-3.86	-4.20
Zα(4)	-499.7	-434.9	-448.5	-472.0	-420.0
Zα(12)	-536.1	-469.6	-488.4	-509.5	-425.1
Zt(4)	-20.27	-21.63	-23.40	-23.26	-24.22
Zt(12)	-19.45	-20.10	-21.29	-21.56	-23.95

The critical values for this table are found in tables 2.2 and 2.5.

Table 2.12

Unit Root Tests with Nonzero Mean and Trend under the Ho Using First Differences of Prices (Model 2.4)

Tests	88(3)	88(6)	88(9)	88(12)	89(3)
DF	-21.41	-21.85	-22.75	-22.40	-21.56
ADF(4)	-8.58	-8.45	-8.39	-8.63	-8.80
ADF(12)	-4.04	-4.22	-3.97	-3.85	-4.21
Zα(4)	-499.5	-435.1	-448.5	-472.0	-419.6
Zα(12)	-534.5	-468.9	-488.2	-509.6	-420.9
Zt(4)	-20.27	-21.61	-23.37	-23.24	-24.33
Zt(12)	-19.47	-20.11	-21.27	-21.53	-24.38

The critical values for this table are found in table 2.3 and 2.6.

In the next section we choose the most appropriate random walk model.

Choosing the Most Appropriate Random Walk Model

Because the random walk will be used in the analyses conducted in the next chapters, an ordinary least squares (OLS) regression is conducted on all five data sets to suggest which of models 2.2-2.4 is the most appropriate. The results from running the regression

$$P_t = \alpha + \beta P_{t-1} + \gamma t + e_t \tag{2.14}$$

are given in table 2.13. The t-statistic given for $\hat{\beta}$ is the statistic $(\hat{\beta}-1)/\hat{\sigma}_\beta$.

Table 2.13

Choosing the Appropriate Random Walk Model I

Data	88(3)	88(6)	88(9)	88(12)	89(3)
$\hat{\alpha}$	1.36 (1.74)	5.14 (3.53)	5.88 (3.64)	1.55 (1.97)	1.48 (1.68)
$\hat{\beta}$	0.99 (-1.19)	0.95 (-3.19)	0.94 (-3.45)	0.98 (-2.39)	0.98 (-2.11)
$\hat{\gamma}$	.00002 (0.42)	.00003 (0.42)	-.00001 (-0.13)	-.00002 (-0.46)	-.00003 (-0.81)

T-statistics are given in parentheses.

In none of the data sets is it suggested to use a random walk with a trend term. Under the premise that futures prices contain a unit root and because the results of table 2.13 indicate in four of the five data sets that $\hat{\beta}$ is significantly different from 1 at conventional levels of significance, I discarded the trend term and the constant term when they were insignificant and estimated the models again by using OLS. These results are given in table 2.14.

Table 2.14

Choosing the Appropriate Random Walk Model II

Data	88(3)	88(6)	88(9)	88(12)	89(3)
$\hat{\alpha}$	------	1.60 (2.02)	1.70 (1.94)	------	------
$\hat{\beta}$	1.00 (0.00)	0.98 (-2.02)	0.98 (-1.95)	1.00 (-0.58)	1.00 (-0.58)

T-statistics are given in parentheses. I also estimated the simple random walk for data sets 88(6) and 88(9) and in neither case could I reject the estimated β as being significantly different from one.

From table 2.14 one can clearly see that the best random walk model is one without drift nor trend term. Because these results are also consistent with another study conducted on financial futures prices (see Goldenberg, 1989), this model is the one that will be used in later analyses.

CHAPTER 3
NONLINEAR DEPENDENCE

Introduction

In the last twenty-five years, solutions to equilibrium asset pricing models have suggested two very different asset pricing functions. Many researchers, beginning with Samuelson (1965), have used these models to propose that asset prices behave as linear martingale processes. The linear martingale process arises out of the assumptions of perfect capital markets and an instantaneous adjustment process or other more specialized assumptions such as the serial independence of dividend growth rates along with constant relative risk aversion (see Ohlson, 1977). Other researchers, Lucas (1978) and Breeden (1979), have shown that general equilibrium asset pricing models are more likely to be consistent with pricing functions that are stochastic and nonlinear if agents are risk averse. Why should nonlinear dependence or departures from linear martingales be so surprising then, when the assumptions under which linear martingale processes hold are, comparatively, so restrictive? Actually they are not surprising, but what is is that theoretical extensions, which incorporate the assumption of risk aversion, have not been able to account for the departures from the martingale process that one sees empirically. Hence, tests and explanations of market performance, based on both linear and nonlinear asset pricing functions are inconclusive.

In the recent past there have been many statistical contributions made to the nonlinear time series literature. With these new contributions, to name a few, the ARCH specification test (Engle, 1982), the ARCH-in-mean specification test (Engle, Lilien, and Robins, 1987), Tsay's test for nonlinear dependence (1986), and the BDS test proposed by Brock, Dechert, and Scheinkman (1987), researchers in both economics and finance have turned again to investigating the statistical properties found in economic and financial data. In studies of financial data, Scheinkman and LeBaron (1989) and Akgiray (1989) found stock returns to be nonlinearly dependent. Hsieh (1989), and Papell and Sayers (1989) found that changes in foreign exchange rates are nonlinearly dependent. In addition, both of the latter papers using changes in exchange rates and the paper by Akgiray (1989) using stock returns found that these data are characterized by effects that may be successfully modelled by ARCH or Generalized ARCH models.

The purpose of this chapter is to test changes in 90-day U.S. Treasury Bill futures prices for nonlinear dependence. In chapter 2, I established that futures prices are nonstationary, that is, that changes in futures prices are stationary. What has not been settled is the process by which these changes evolve. If changes are independent, then the random walk model can be justified; if not, then nonlinear dependence must be considered when modelling the data.

In the next section, I investigate whether or not changes in the contract's price are independent. Several tests, both parametric and nonparametric, are used for this purpose. Next, in the section entitled "A More Powerful Test of Dependence," a brief account of the Brock, Deechert, Scheinkman (1987), hereafter BDS, statistic is given and then used to detect any dependence that the data may contain. In the section following this one, three tests for nonlinearity are discussed and then conducted, namely, Tsay's (1986) test, a specification test for Engle's (1982) autoregressive conditional heteroskedasticity (ARCH) model, and the BDS test applied to filtered data. These tests are jointly used since Tsay's test has good power against nonlinear moving average processes and bilinear models, ARCH processes should be detected by the ARCH specification test, and if other types of nonlinear dependence exist, then the BDS test, being a more general test, should recognize them. In the penultimate section I explain and differentiate between the types of nonlinearity that may describe the data. For this purpose I use Hsieh's (1989) test. The summary completes the chapter.

Testing for Independence

The methods used are both parametric and non-parametric tests based on the time domain. Namely, the analysis involves the parametric Box-Pierce Q test and the non-parametric difference-sign test, turning point test, and a test based on a mixed statistic. Tests involving spectral analysis or filters present complexities and therefore are avoided (see Praetz, 1976).

Q refers to the Box-Pierce statistic and is used to test the assumption of white noise disturbances. The alternative hypothesis under this test, namely non-white noise disturbances, might seem rather vague, however the Q-test can be derived as the Lagrange Multiplier test against AR(p) or an MA(p) process. It is distributed $\chi^2(k)$ where k is the lag. K, the statistic of the difference-sign test, is the number of + signs in the sequence e_t and is asymptotically distributed normally with mean (N-1)/2 and variance (N+1)/12 where N is the length of the sequence. If a sequence is independent with mean zero, then the number of positive values should not be significantly different from the amount of negative ones. Hence, the K-statistic measures this departure from independence. The turning point test statistic, r, refers to the total number of runs up or down. As opposed to the K-statistic, this statistic measures departures from independence by considering the sequence of positive and negative values. It can be the case that there are as many positive as negative values, but there are only two runs. r is also asymptotically distributed normally with mean (2N-1)/3 and variance (16N-29)/90. The mixed test, sometimes known as a u-run statistic, is formed by combining the statistics of the difference-sign and turning points tests. The statistic is constructed as follows:

$$T_{K,r} = Z_K^2 + Z_r^2 \qquad (3.1)$$

where the Z's are standardized forms of K and r respectively. The limiting distribution of $T_{K,r}$ is $\chi^2(2)$ since K and r are asymptotically independent. The intention of the u-run statistic is to make the test less sensitive to specific patterns and more sensitive to general departures from independence.

The results from applying these tests to the series e_t are presented in Table 3.1.

Table 3.1

Tests For Independence

Contracts	88(3)	88(6)	88(9)	88(12)	89(3)
Q (lag 6)	11.77	3.87	10.22	13.34*	14.92**
Q (lag 12)	14.58	10.50	16.05	17.40	21.49*
Q (lag 18)	17.51	11.80	18.68	20.47	23.77
Q (lag 24)	20.10	17.02	24.49	26.36	27.65
Z_K	-.55	-.08	-.34	-.40	.32
Z_r	-2.39**	-.46	-.86	-1.24	-1.65
$T_{K,r}$	6.01*	.22	.86	1.70	2.86

Z_K and Z_r refer to the standardized forms of K and r respectively. The null hypothesis under all tests is that successive price differences are random. * and ** denote a rejection of the H_0 at the 5% and 2.5% levels of significance respectively.

By looking at table 3.1, it is reasonable to accept that price changes for any of the five contracts are independent. This is not surprising for several reasons. For one, these tests are not very powerful. Secondly, unless the distribution of price changes is normal, these tests can only verify that these changes are uncorrelated, not statistically independent. Thirdly, time series generated by nonlinear moving average models, threshold autoregressive models, bilinear time-series models, or ARCH models exhibit little or no serial correlation even though the time series may be statistically dependent across time. Because of these reasons and the possibility that nonlinear asset pricing equations may provide a better description of the evolution of some asset prices, these time series are tested for nonlinearities in the third section.

A More Powerful Test of Dependence: The BDS Test.

The BDS test, suggested in a paper by Brock, Dechert, and Scheinkman (1987), also is designed to detect departures from independence. This test, however, as compared to the tests discussed above, is more powerful. It has the power to recognize dependencies in underlying processes that are nonlinear as well as linear. As compared to other well known tests of nonlinear dependence, such as the ARCH specification test, the BDS test is more general. It is able to discern nonlinearities that are often not found when other tests, which target specific types of nonlinearity, are used.

Formally, the BDS statistic is constructed by using the correlation integral. The correlation integral is defined as

$$C_d(\delta, T) = \frac{2}{N^2 - N} \sum_{i<j} I_\delta(e_i^d, e_j^d) \qquad (3.2)$$

where $I_\delta(x,y) = 1$ if $|x,y| < \delta$ and 0 otherwise, $|x,y| = \max_j |x^j, y^j|$, δ is the tolerance distance chosen by the researcher, d is the embedding dimension, $N = T - d + 1$, and T is the length of the time series. It is used to calculate the number of d-histories whose distances from one another is less than the chosen value δ. Consider the series of successive price differences e_t. If this series has length T, then it is possible to create $N = T-(d-1)$ subseries of length d. If we denote these subseries, or d-histories, by $\{e_t^d\}$, where $\{e_t^d\} = \{e_t, e_{t+1}, \ldots, e_{t+d-1}\}$, then the correlation integral can be used as a measure of clustering. If the subseries cluster in any dimension, then the correlation integral will take on relatively larger values. From this premise, BDS (1987) formed their statistic. If under the null hypothesis e_t is independently and identically distributed random variable, BDS (1987) showed that the quantity $D_d = \{C_d(\delta)-[C_1(\delta)]^d\}$ should approach 0 as $T \to \infty$. They also show that under the same null, the statistic

$$BDS(d, \delta) = T^{\frac{1}{2}} \frac{D_d}{b_d} \qquad (3.3)$$

where b_d is the consistent estimate of the standard deviation of the statistic $T^{1/2}D_d$ (for an exact value for this standard deviation see Hsieh, 1989), converges to a N(0,1) variable as $T \to \infty$. For large values of the BDS(d,δ) statistic, the null hypothesis, as stated above, is rejected.

The finite sample properties of the BDS statistic are discussed in Hsieh and LeBaron (1988). To obtain the size of the statistic under the null hypothesis, they generated pseudo-random numbers for the following distributions: 1) Standard Normal, 2) Student-t with 3 degrees of freedom, divided by $\sqrt{3}$, 3) Double exponential distribution, divided by $\sqrt{2}$, 4) Chi-Square with 4 degrees of freedom, divided by $\sqrt{8}$, 5) Uniform on $(0,2\sqrt{3})$, 6) Bimodal mixture of normals: .5 N(3,1) + .5 N(-3,1), divided by $\sqrt{10}$. For the sample size closest to the sample sizes considered in this paper, T=500, they considered embedding dimensions, d, of 2 through 6 for each of these distributions. Using their results, the value of δ is kept between 1 and 2 times the standard deviation of the data. It is only between these values of δ that the statistic's finite sample distribution, under all six distributions, remains reasonably close to its limiting distribution.

Table 3.2

The BDS Test Statistics for Daily Price Changes in Contract 88(3)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	3.9889	4.2956	4.2732	4.2292	4.1517
3	4.6422	4.9603	5.0371	5.1754	5.5658
4	5.3913	5.5302	5.3738	5.4578	5.9125
5	6.1221	6.1966	5.9142	5.9678	6.5182

SD denotes the standard deviation of the sample containing the data for contract 88(3). The standard deviation for this series is 0.1077. Each table below, 3.3-3.6, will use their corresponding standard deviation. So SD in table 3.3 corresponds to the standard deviation of the sample containing the data for contract 88(6).

Table 3.3

The BDS Test Statistics for Daily Price Changes in Contract 88(6)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	5.2652	5.4226	6.3720	8.3590	9.1464
3	5.5696	5.5757	6.0939	7.7403	8.2232
4	6.0014	5.8263	6.3881	8.1974	8.9037
5	6.4692	6.2700	6.7993	8.5717	9.1674

See note above. The standard deviation for this series is 0.1121.

Table 3.4

The BDS Test Statistics for Daily Price Changes in Contract 88(9)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	3.0903	3.4694	3.1709	3.2467	3.9871
3	3.2882	3.7190	3.4700	3.3861	3.7738
4	3.9425	4.3936	4.1401	4.1259	4.8387
5	4.3941	4.9232	4.7071	4.6220	5.3392

See note above. The standard deviation for this series is 0.1071.

Table 3.5

The BDS Test Statistics for Daily Price Changes in Contract 88(12)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	4.0408	4.7484	5.2530	6.7436	8.8590
3	4.0633	4.8537	5.4303	6.2574	7.5480
4	4.4454	5.2891	5.8382	6.6060	8.0152
5	5.2774	6.0475	6.4174	7.1947	8.7382

See note above. The standard deviation for this series is 0.1086.

Table 3.6

The BDS Test Statistics for Daily Price Changes in Contract 89(3)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	1.9596	2.8260	3.7682	4.5710	4.9149
3	2.7427	3.5656	4.3150	4.7623	4.7898
4	3.4330	4.3962	5.2114	5.8230	6.2462
5	4.0287	4.9942	5.7441	6.2903	6.7551

See note above. The standard deviation for this series is 0.1033.

In tables 3.2-3.6 results from the BDS tests are given. The BDS statistic strongly suggests that dependence is apparent in every data set under investigation. The statistic is significant at the 1% level for each contract, the critical value being 2.576, under all embedding dimensions, and for all sizes of δ chosen, except for d=2 and δ=1x(standard deviation) in the fifth contract. Though these results may seem quite strong, they are consistent with other studies of financial data. Hsieh (1989), in his investigation of changes in exchange rate data, also finds that the BDS statistics are extremely significant for all exchange rates, under all embedding dimensions, and for all sizes of δ chosen. Scheinkman and LeBaron (1989), by means of the BDS statistic, also report that weekly stock returns are not i.i.d..

Tests for Nonlinear Dependence

In this section three tests for nonlinear dependence are conducted. The ARCH specification test is constructed in the familiar way. Under the null hypothesis, the squared residuals are assumed to be white noise. The test statistic is formulated by regressing the squared daily price changes on its lags and calculating the LM test statistic $N * R^2$ where N is the number of observations. The statistic is distributed $\chi^2(p)$ where p is the number of lags in the regression.

Tsay's test, under the null hypothesis, assumes that daily price changes are i.i.d.. Simulations in Tsay (1986) show that this test has good power against nonlinear moving average and bilinear models. The test statistic is constructed in the following way:

1) regress e_t on a constant and lags $e_{t-1},\ldots,e_{t-J}$ and save the residuals u_t
2) regress e^2_{t-1}, $e_{t-1}e_{t-2}$, $e^2_{t-2},\ldots,e^2_{t-J}$ on the same lags as in 1), $e_{t-1},\ldots,e_{t-J}$, and save the residual vector Z_t
3) regress u_t on Z_t and save the residual v_t
4) form the test statistic in the following way:

$$\psi = \frac{\{[\sum_t Z_t u_t]\,[\sum_t Z_t' Z_t]^{-1}[\sum_t Z_t' u_t]/j\}}{\{\sum_t v_t^2/[N-J-j-1]\}} \sim F(j,N-J-j-1) \qquad (3.4)$$

where N is the number of observations, J is dimension of the lag in 1) and 2), and j = J(J+1)/2. The limiting distribution of the test statistic is F(j,N-J-j-1).

The results from applying the ARCH specification test and Tsay's test to daily price changes are given in tables 3.7 and 3.8 respectively. According to the ARCH specification test, the data exhibits extreme multiplicative dependence for lags 3, 4, and 5. Nonlinear dependence also appears in the results of the Tsay test.

Table 3.7

ARCH Specification Tests

Contracts	88(3)	88(6)	88(9)	88(12)	89(3)
lag 1	1.79	1.28	1.71	3.45	2.78
lag 2	6.43	2.20	1.72	4.55	2.75
lag 3	9.08	12.59*	27.17*	38.03*	51.86*
lag 4	24.04*	30.09*	38.35*	53.62*	58.24*
lag 5	24.00*	33.47*	38.35*	55.51*	58.23*

The asterisk in tables 3.7 and 3.8 indicates rejection of the null at the 1% level of significance.

Table 3.8

Tsay's Tests for Nonlinearity

	J=2	J=3	J=4	J=5
Contract 88(3)	2.309	3.965*	5.927*	4.034*
Contract 88(6)	1.221	4.219*	5.751*	4.136*
Contract 88(9)	.868	-.361	4.733*	3.439*
Contract 88(12)	2.260	3.263*	5.519*	3.816*
Contract 89(3)	1.662	3.052*	5.244*	3.186*

These tests above, the ARCH specification test and Tsay's test, have good power against deviations from their specific nulls, but, they, like any statistical test, are not foolproof. To lend emphasis to these results the BDS test is also applied. As mentioned above, the BDS test is a general test of clustering. It distinguishs random from nonrandom behavior by considering whether or not a time series clusters in dimensions greater than one. However, because the BDS test may capture linear as well as nonlinear dependence, before I can apply this test, the data are first purged of any linear dependence[1]. To do this e_t is regressed on 10 of its lags and the residuals are then kept for observation. The estimated coefficients and their corresponding t-statistics from these regressions are given in table 3.9.

The results, on the whole, are not surprising. Besides the fact that in three of the contracts the third lag has a significant t-statistic and in one contract the second lag does at the 5% level of significance, the data cannot be explained by its lags.

[1] Brock (1987) has shown that the asymptotic distribution of the BDS test applies to residuals of linear regressions.

Table 3.9

Regression Results From a Linear Purge of the Data

Contract	88(3)	88(6)	88(9)	88(12)	89(3)
constant	.120 (.240)	.011 (.021)	-.358 (-.663)	-.312 (-.645)	-.242 (-.507)
lag 1	.043 (.909)	.013 (.266)	.010 (.202)	-.019 (-.398)	.002 (.032)
lag 2	.150* (3.20)	.071 (1.46)	.028 (.559)	.037 (.778)	-.056 (-1.18)
lag 3	-.015 (-.324)	-.065 (-1.33)	-.128* (-2.53)	-.139* (-2.97)	-.147* (-3.12)
lag 4	-.062 (-1.31)	.0096 (.196)	.027 (.525)	.0063 (.133)	.055 (1.16)
lag 5	-.015 (-.327)	-.048 (-.977)	-.037 (-.723)	.001 (.211)	-.032 (-.667)
lag 6	.066 (1.40)	.060 (1.23)	.073 (1.44)	.061 (1.29)	.066 (1.39)
lag 7	.044 (.923)	.080 (1.64)	.075 (1.48)	.043 (.897)	.039 (.828)
lag 8	-.069 (-1.47)	-.082 (-1.67)	-.069 (-1.37)	-.047 (-.995)	-.094* (-2.00)
lag 9	.121 (.259)	.048 (.983)	.060 (1.18)	.052 (1.10)	.023 (.481)
lag 10	.370 (.789)	.063 (1.36)	.043 (.851)	.056 (1.24)	.015 (.328)
R^2	.033	.031	.037	.034	.044

T-statistics are in parentheses. The asterisk indicates significance at the 5% level of significance.

In tables 3.10-3.14, the results from applying the BDS test to these residuals, or linearly purged daily price changes, are given. As compared to tables 3.2-3.6, the BDS statistics are not as significant and for several embedding dimensions and sizes of δ, especially in contracts 88(9) and 89(3), the test statistic, using the same critical value as that above, is insignificant at the 1% level of significance. Nevertheless, nonlinear dependence is still an apparent part of all the contracts when the data is considered in the third, fourth, and fifth dimensions.

Table 3.10

The BDS Test Applied to the Linearly Purged Contract 88(3)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	3.5623	3.7048	3.6264	3.5316	3.5157
3	4.1828	3.9711	3.4567	3.1463	3.0836
4	4.9650	4.6282	3.9915	3.6501	3.6135
5	5.7971	5.3844	4.6339	4.2265	4.3420

In tables 3.10-3.14 the BDS test is applied to the filtered data produced by removing any autocorrelative structure. SD refers to the standard deviation of the purged contract. SD for this series is 0.1068.

Table 3.11

The BDS Test Applied to the Linearly Purged Contract 88(6)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	3.3957	3.6894	3.4385	3.2187	3.0684
3	3.9326	4.3487	3.9327	3.4878	3.1275
4	4.8076	5.3116	4.8022	4.2651	3.9466
5	5.4210	6.0502	5.5844	5.0139	4.7938

See note above. The standard deviation for this series is 0.1048.

Table 3.12

The BDS Test Applied to the Linearly Purged Contract 88(9)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	2.4435	2.6273	2.2894	2.2408	2.6197
3	2.8618	3.1556	3.0697	2.8215	2.9288
4	3.6944	4.1983	4.2096	3.8941	3.9861
5	4.2409	4.9140	5.0146	4.6610	4.7389

See note above. The standard deviation for this series is 0.1059.

Table 3.13

The BDS Test Applied to the Linearly Purged Contract 88(12)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	2.5457	2.8968	2.9404	2.8035	2.2895
3	2.7290	3.2339	3.5171	3.4107	3.2920
4	3.4815	4.1733	4.4914	4.4012	4.3097
5	4.1724	5.0279	5.4630	5.3847	5.2115

See note above. The standard deviation for this series is 0.1025.

Table 3.14

The BDS Test Applied to the Linearly Purged Contract 89(3)

d	δ=1xSD	δ=1.25xSD	δ=1.5xSD	δ=1.75xSD	δ=2xSD
2	.2148	.9757	1.8349	2.4199	2.4109
3	.7160	1.5176	2.3759	2.9271	2.8889
4	1.8713	2.6158	3.4325	3.9639	4.2048
5	2.6073	3.3522	4.1733	4.7390	4.9973

See note above. The standard deviation for this series is 0.1.

Differentiating Between Additive and Multiplicative Nonlinear Dependence

In this section I examine this nonlinear dependence a little closer. Several theoretical models have recently been proposed to handle nonlinear time series. Some of the more popular models are the following:

Robinson (1979) suggested the Nonlinear Moving Average (MA) model. A simple example is

$$e_t = u_t + \alpha u_{t-1} u_{t-2}. \tag{3.5}$$

Tong and Lim (1980) introduced the threshold autoregressive model. An example is

$$\begin{aligned} e_t &= \alpha e_{t-1} + u_t, \text{ if } e_{t-1} \leq 1, \\ e_t &= \beta e_{t-1} + u_t, \text{ otherwise.} \end{aligned} \tag{3.6}$$

Granger and Andersen (1978) proposed the bilinear time-series model.

$$e_t = u_t + \alpha e_{t-1} u_{t-1}. \tag{3.7}$$

In all three models above, e_t is the daily price change, u_t is a normal, independently and identically distributed random variable with mean 0 and variance σ^2, $E(e_t/e_{t-1}, u_{t-1}) \neq 0$ and $Var(e_t/e_{t-1}, u_{t-1}) = \sigma_2$.

A modest example of Engle's (1982) Autoregressive Conditional Heteroskedastic (ARCH) model is

$$e_t = u_t \tag{3.8}$$

where u_t is conditionally normally distributed with mean 0 and variance

$$h_t = [\alpha_0 + \alpha_1 e^2_{t-1}]. \tag{3.9}$$

The time-varying parameter (TVP) model, of which a simple example is

$$e_t = \beta_t e_{t-1} + u_t \tag{3.10}$$

$$\beta_t = \alpha + \delta z_t + v_t$$

where z_t is some variable that explains movements in β_t, $Var(u_t) = \sigma^2_u$, $Var(v_t) = \sigma^2_v$, $Cov(u_t,v_t) = 0$ for all t and s, and $Cov(u_t,u_s) = Cov(v_t,v_s) = 0$, for all $t \neq s$.
In both of these models $E(e_t/e_{t-1},u_{t-1}) = 0$ and $Var(e_t/e_{t-1},u_{t-1})$ is not constant over time.

The data contains at least one of two types of nonlinear dependence, additive and/or multiplicative.

Additive dependence:

$$u_t = v_t + f(e_{t-1},\ldots,e_{t-k},u_{t-1},\ldots,u_{t-k})$$

Multiplicative dependence:

$$u_t = v_t f(e_{t-1},\ldots,e_{t-k},u_{t-1},\ldots,u_{t-k})$$

where v_t is an i.i.d. random variable with zero mean and independent of past e_t's and u_t's, e_t's are the price differences, u_t are the residuals from the linear regression results given in table 3.9, and f( ) an arbitrary nonlinear function of $e_{t-1},\ldots,e_{t-k},u_{t-1},\ldots,u_{t-k}$, for some finite k. We differentiate between these types by looking at the data's conditional means and variances. If the data solely exhibits additive nonlinear dependence, then the dependence enters only through the mean of the process and the conditional mean and variance will be similar to those expressed by the first three models above. If the data solely displays multiplicative nonlinear dependence, then the dependence enters only through the variance of the process and the last two models would be candidates for modelling their evolution.

Hsieh (1989) developed a test to distinguish between these two types of nonlinear dependence that is based on examining the conditional mean and variance of a time series. The time series, the first differences of daily prices, is first purged of linear dependence by using the residuals from the regression results given in table 3.9. The test is defined in the following way.

$p_{vvv}(i,j)$ is defined as $E(v_t, v_{t-i}, v_{t-j})/p_v^3$, where v_t are the residuals from the regression equations in table 3.9. The null hypothesis is that the process contains multiplicative nonlinearity. Note that this implies that $E(v_t, v_{t-i}, v_{t-j})/p_v^3=0$ for all $i,j > 0$. $p_{vvv}(i,j)$ is estimated by

$$r_{vvv}(i,j) = \frac{1/T \sum_{t=j>i}^{T} v_t v_{t-i} v_{t-j}}{[1/T \sum_{t=1}^{T} v_t^2]^{3/2}} \tag{3.11}$$

Under the null hypothesis, $p_{vvv}(i,j) = 0$ and $\sqrt{T}[(1/T)\Sigma\ v_t, v_{t-i}, v_{t-j}]$ is asymptotically normally distributed with mean 0 and variance $w_{i,j} = \operatorname*{plim}_{T\to\infty} (1/T)\Sigma\ v_t^2, v_{t-i}^2, v_{t-j}^2$ provided that the probability limit exists. Given this, $r_{vvv}(i,j)$ is asymptotically distributed $N(0, w_{i,j}/\sigma_v^6)$. $w_{i,j}/\sigma_v^6$ can be consistently estimated by

$$[(1/T)\Sigma\ v_t^2, v_{t-i}^2, v_{t-j}^2]/[(1/T)\Sigma\ v_t^2]^2. \tag{3.12}$$

The third-order moment test, as Hsieh (1989) calls it, is designed to reject the null hypothesis only in the presence of additive nonlinear dependence. The test statistic is

$$H_{i,j} = \sqrt{T}\, r_{vvv}(i,j)/[w_{i,j}/\sigma_v^6]^{1/2}. \tag{3.13}$$

A rejection of the null for a two-tailed test at the 1% level of significance is found if the absolute value of $H_{i,j}$ is larger than 2.576. The test is applied to the futures data for p of 5 lags for both i and j. The results are presented in table 3.15 below.

These results indicate that multiplicative dependence is the type found in the data for all contracts except 88(9). Since the null is rejected for every lag except (1,1) when testing contract 88(9), I conclude that there exists additive nonlinearity in this data set. However, the results from the ARCH specification test above suggests that multiplicative dependence is also a part of this data set. Therefore, I conclude that all data sets contain at least multiplicative dependence and some may contain additive dependence.

Table 3.15

Hsieh's Test to Distinguish Betweeen Additive and Multiplicative Nonlinear Dependence

Lags i,j	Contracts: 88(3)	88(6)	88(9)	88(12)	89(3)
1,1	.395	.308	2.572	.280	.979
2,1	.521	.764	10.720	1.601	.926
2,2	.958	.832	5.864	.552	.550
3,1	1.680	1.563	14.638	1.235	.804
3,2	.696	.839	9.740	1.328	.726
3,3	.627	.776	7.353	.764	.810
4,1	1.585	1.554	14.914	1.475	1.542
4,2	1.039	1.326	14.967	1.600	.759
4,3	1.064	.740	8.732	1.183	1.144
4,4	1.195	1.195	11.750	1.155	1.207
5,1	.813	.459	4.007	.699	.272
5,2	1.817	.960	7.785	.764	.598
5,3	1.234	.996	8.495	.790	.304
5,4	.655	.493	6.693	.754	.945
5,5	.876	1.016	10.856	.895	1.132

Note: The results presented are the absolute values of the test statistics.

Summary

There is mounting evidence, Hsieh (1989), Scheinkman and LeBaron (1989), and Papell and Sayers (1989), that asset-prices contain nonlinear dependencies that are not modelled by linear asset-pricing functions nor considered by asset traders.

This chapter shows that dependencies are found in the changes of futures prices. Evidence from the BDS test, Tsay's test for nonlinearity, and the ARCH specification test indicates that this dependence is nonlinear. Hsieh's test, the third-order moment test, suggests that the nonlinear dependencies are primarily found in the variances of the data.

CHAPTER 4
PREDICTION

Introduction

In chapter 2, using several tests for nonstationarity, I concluded that Treasury Bill futures prices contain a unit root. In chapter 3, first differences of the data were tested for nonlinear dependence. Relying on the results from several tests, an part of the behavior of Treasury Bill futures prices appears to be explained by nonlinear dependence. Hsieh's (1989) test indicated that the type of nonlinear dependence is multiplicative, i.e., the nonlinearity enters through the variance of the process. However, when using Tsay's (1986) test, which has good power against processes that contain additive nonlinear dependence (nonlinear dependence that enters through the mean of the process), nonlinear dependence was found. Hence, it is very possible that the data contain both additive and multiplicative nonlinear dependence.

In this chapter the issues of modelling nonlinear time series and nonlinear prediction are addressed. Because of the results in the previous chapter, the data are presumed to contain nonlinear dependencies. Hence, I first model the data using several nonlinear models and then compare them against a simple random walk process and each other by considering their predictive power.

So far, this concern has not been addressed in the futures markets' literature, but recently, in the exchange rate literature, this issue has emerged. The conclusions are mixed and depend upon the particular models used to capture the nonlinearities. Meese and Rose (1989) find that the poor explanatory power that several popular exchange rate models exhibit cannot be attributed to nonlinearities arising from time deformation or improper functional form. Diebold and Nason (1990) nonparametrically estimate the conditional mean functions of ten major exchange rates using a technique known as "locally weighted regression." They conclude that considering nonlinearities does not help point prediction. On the other hand, by modelling exchange rate dynamics as a sequence of stochastic, segmented time trends, Engel and Hamilton (1990) find that nonlinear dependence may be exploitable for predictive purposes. They show that stochastic, segmented trends model predicts better than a random walk.

In this chapter, I compare the predictions of a simple random walk process, i.e.,

$$P_t = P_{t-1} + e_t \text{ , where } e_t \sim N(0,\sigma^2) \qquad \textbf{(4.1)}$$

with those of an ARCH-in-Mean (ARCH-M) model (Engle, Lilien, and Robins, 1987), Generalized Autoregressive Conditional

Heteroskedastic-in-Mean (GARCH-M) model (Bollerslev, 1986), the bilinear model (Granger and Andersen, 1978a), a Time Varying Parameter. model (TVP), a Time Series Segmentation Model (Sclove, 1983), and a Stochastic, Segmented Trends model (Hamilton, 1989) below.

The chapter is organized as follows. The next section describes in detail and justifies each of the nonlinear models considered. After this discussion, a section is devoted to fitting the models to the data. Some diagnostic tests on the residuals of the estimated models are also conducted in this section. Prediction and comparison of the models are taken up after this and the last section summarizes the chapter.

A Look at Some Nonlinear Time Series Models

Six nonlinear models are considered in this section. These models are the ARCH-in-Mean (ARCH-M) model (Engle, Lilien, and Robins, 1987), Generalized Autoregressive Conditional Heteroskedastic-in-Mean (GARCH-M) model (Bollerslev, 1986), the bilinear model (Granger and Andersen, 1978a), a Time Varying Parameter model (TVP), a Time Series Segmentation Model (Sclove, 1983), and a Stochastic, Segmented Trends model (Hamilton, 1989).

The ARCH-M and GARCH-M Models

The ARCH-M and GARCH-M models both use a function of the conditional variance of a time series to explain the mean of its process. Consider the series P_t to be modeled as

$\Delta P_t = \delta + \gamma h_t^{1/2} + e_t$, where e_t is an error term with zero mean and conditional variance $h_t = E(e_t^2 | I_{t-1})$ and I_{t-1} is the set of all information available at time t. A specific form of the conditional variance

$$h_t = \alpha_0 + \sum_{k=1}^{q} \alpha_k e_{t-k}^2 \quad , \tag{4.2}$$

proposed by Engle, Lilien, and Robins (1987), is known as the ARCH-M(q) model. Bollerslev (1986) generalized this form by allowing lagged values of the conditional variance, in addition to lagged squared residuals, to explain its contemporaneous value, i.e.,

$$h_t^{\bullet} = \alpha_0 + \sum_{j=1}^{p} \beta_j h_{t-j} + \sum_{k=1}^{q} \alpha_k e_{t-k}^2 \quad . \tag{4.3}$$

When this form is used the model is known as the generalized ARCH-M model or GARCH-M(p,q) model. The parameters of both models satisfy the following conditions when appropriate: $\alpha_0 > 0$, α_k, $\beta_j \geq 0$, k=1,...,q, j=1,...,p.

The empirical distribution of the variables generated by these processes are heavy tailed, compared to the normal distribution. The unconditional mean and variance of an ARCH-M and GARCH-M process are constant, equal to

$$\frac{\alpha_0}{(1 - \sum_{k=1}^{q} \alpha_k)} \tag{4.4}$$

respectively, but the conditional mean and variance are time

$$\frac{\alpha_0}{(1 - \sum_{j=1}^{p} \beta_j - \sum_{k=1}^{q} \alpha_k)} \tag{4.5}$$

dependent as shown above. The fact that conditional variances are allowed to depend on past realized variances is consistent with the actual volatility pattern observed in most financial markets during both stable and unstable periods.

ARCH and GARCH models have been sucessfully applied to foreign exchange rate data by Domowitz and Hakkio (1985), Diebold and Pauly (1988), and Hsieh (1989), and to stock market data by Akgiray (1989). Engle, Lilien, and Robins (1987) fruitfully applied the ARCH-M to expected bond returns and Engle and Bollerslev (1986) used a GARCH-M model to model the risk premium on the foreign exchange market.

The Bilinear Model

The bilinear model, proposed by Granger and Andersen (1978a), was introduced as a simple generalization to linear models. This class of nonlinear models may be regarded as the natural nonlinear extension to Autoregressive Moving Average (ARMA) processes. Just as the ARMA process is sufficiently general to approximate most linear series that arise in the real world, the introduction of the bilinear model marked the beginning of work in time series analysis concerned with finding a general nonlinear, univariate model. The bilinear model is not dramatically nonlinear, however the bilinear class of models are non-explosive and invertible and useful in forecasting. Granger and Andersen (1978a) applying simple bilinear models to IBM daily common stock closing prices[1] and Gabr and Rao (1981) applying a bilinear model to Canadian Lynx data[2] both show the ability of the bilinear model to forecast. Maravall (1983) shows that bilinear models are able to improve upon the Bank of Spain's linear ARIMA forecasts of currency demand.

In this chapter I consider a specific first-order bilinear model motivated by the following:
In futures markets literature, it has long been accepted that futures prices follow a simple random walk process. That is

$$P_t = P_{t-1} + e_t \text{ , where } e \sim N(0, \sigma^2) .$$

Although I concluded that Treasury Bill futures prices contain a unit root, some doubt was cast on this specification in chapter 3. There, it was shown that e_t exhibits nonlinear dependence if P_t is modeled as a random walk. This leads me to believe that the specification may be more reasonable if the expectation of P_t at time t-1 is permitted to be a nonlinear function of past information.

If a series P_t is generated by

$$P_t = (\text{expectation of } P_t \text{ made at time t-1}) + e_t$$

so that e_t is essentially the expectation error, and if these

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[1] These are closing prices for 169 trading days beginning May 17, 1961.

[2] This data gives the number of lynx trapped annually rather than the actual population.

expectations are a function of the most recent data available at time t-1, that is P_{t-1} and e_{t-1}, of the form

$$E(P_t | P_{t-1}, e_{t-1}) = g(P_{t-1}, e_{t-1})$$

then there is no reason to believe that this function will be linear. One way of picking up at least part of the nonlinearity is to use the approximation

$$g(P,e) = aP + bPe + de$$

which gives a bilinear model for the series P_t.

Note that the specified approximation allows for both "main effects," aP and de, and an "interaction" or "cross-impact" effect bPe. The first-order bilinear model that results for futures prices is

$$P_t = aP_{t-1} + bP_{t-1}e_{t-1} + de_{t-1} + e_t, \tag{4.6}$$

where e_t is the usual white noise series.

A Time-Varying Parameter Model

In another attempt to model nonlinearity a simple time-varying model is used. The intuition behind its use is that if a market is not fully efficient, as I concluded about the Treasury Bill futures market in chapter 3 due to the nonlinear dependence found in the data, then not all of a market's relevant information will be disclosed by its price. If the price of a security does not contain all of the market's relevant information, then dependent upon the importance of the information not contained in the price, it is possible that the reaction of today's price to yesterday's will be different across time. Hence, I propose the following model:

$$P_t = \beta_t P_{t-1} + \epsilon_t \text{ , where } \epsilon_t \sim i.i.d.(0,\sigma^2) \qquad (4.7)$$

$$\beta_t = \alpha\beta_{t-1} + u_t \text{ , where } u_t \sim i.i.d.(0,\theta^2)$$

The equation representing the evolution of β_t reflects a learning process on the part of market participants.

A Time-Series Segmentation Model

In addition to the models proposed above, there are other paradigms that are specifically used for nonlinearly dependent variables and may explain the nonlinear dependence found in futures prices. One model that can be easily imagined is a time-series segmentation model which hypothesizes that the changes in prices conform to one of two processes where the processes are dependent upon particular states of nature. An explanation for why this model may be appropriate for futures prices is the same as the explanation given for the appropriateness of TVP model though in the segmentation model the dependence of today's price on yesterday's is more systematic.

The model that I use is a specific form of the model that Sclove (1983) proposed. I assume that there are two states of the world, γ=1,2, and the changes in futures prices follow a second-order autoregressive process under each state, i.e.,

$$\Delta P_t = \alpha_\gamma \Delta P_{t-1} + \beta_\gamma \Delta P_{t-2} \quad .$$

Changes in futures prices are modeled because an assumption of the model is that the variable under analysis is covariance-stationary. The first-order autoregressive parameter is assumed to be positive and negative under states 1 and 2 respectively and the second-order parameter is not constrained to be either positive or negative under either state. The model assumes that the residuals from the autoregressive processes under each state are normal processes with constant and equal variances between states.

The algorithm begins by setting initial values of each of the autoregressive processes and setting the transition probabilities, p_{cd}, where c and d indicate the previous and current state respectively, equal to 1/2. Also, the probability that the initial state of the world, $f(\gamma_1)$, is state 1 is set to 1/2. With these initial values, the first state of the world is estimated by maximizing $f(\gamma_1)f(P_1|\gamma_1)$, where

$$f(P_1|\gamma_1=c) = (2\pi\sigma^2)^{-1/2} \exp(-\frac{u_{tc}^2}{2\sigma^2}) \tag{4.8}$$

and

$$u_{tc} = P_t - (\phi_c P_{t-1} + \delta_c)$$

if estimating the first-order autoregressive case. From the second state onward the states of the world are estimated by maximizing

$$p_{\gamma_{t-1}\gamma_t} f(P_t|\gamma_t) \,. \tag{4.9}$$

Once the states are labeled, the parameters from the autoregressive processes can be estimated by separating the data according to the two states and maximizing the likelihood function:

$$L = p_{11}^{j} p_{12}^{k} p_{21}^{l} p_{22}^{T-j-k-l} (2\pi\sigma^2)^{-(T-1)/2} \exp(-\frac{q}{2\sigma^2}) \tag{4.10}$$

where

$$q = \sum_{\gamma_t=1} [P_t - (\phi_1 P_{t-1} + \delta_1)]^2 + \sum_{\gamma_t=2} [P_t - (\phi_2 P_{t-1} + \delta_2)]^2 .$$

The transition probabilities can be estimated by n_{cd}/n_c, where n_{cd} indicates the number of times the state changes from state c to state d and n_c indicates the number of times the process is labeled by state c. With new transition probabilities and new autoregressive parameter estimates, the states, γ, may be reestimated. Finally when no observation changes labels from the previous iteration, the algorithm stops.

The estimation procedure is based on what the likelihood function would have been if states were observable. Implicitly then, it is assumed that the actual historical states of nature are those that maximize the joint likelihood of the changes in futures prices and states which produce the prices.

A Stochastic, Segmented Trends Model

Hamilton (1989) introduced another approach to modeling changes in regime. The stochastic specification is similar to that explored by Sclove (1983) above, although the statistical approach is quite different. The general idea is to decompose a nonstationary time series into a sequence of stochastic, segmented trends. The model postulates the existence of an unobserved state or regime variable, S_t, that is presumed to depend on past realizations of ΔP_t and S only through S_{t-1}. When $S_t=0$, the observed change in the futures price is presumed to have been drawn from a $N(\mu_0,\sigma_0^2)$ distribution, whereas when $S_t=1$, ΔP_t is distributed $N(\mu_1,\sigma_1^2)$; thus when $S_t=0$, the trend in the futures price is μ_0, whereas when $S_t=1$, the trend is μ_1. Discrete shifts in the parameters of the distribution of futures prices are viewed as the outcome of a first-order, discrete-state Markov process which governs the transition between states,

$$\begin{aligned} p(S_t=0 \mid S_{t-1}=0) &= p_{11} \\ p(S_t=1 \mid S_{t-1}=0) &= 1-p_{11} \\ p(S_t=0 \mid S_{t-1}=1) &= 1-p_{22} \\ p(S_t=1 \mid S_{t-1}=1) &= p_{22}. \end{aligned} \qquad \textbf{(4.11)}$$

Given the parameters of the distribution, where it is assumed that the first and second moments completely describe the distribution, and a Markov process describing the transition probabilities from one state of nature to another, the state to which the segment of the series belongs is determined.

Hamilton's statistical approach differs from Sclove's in that the actual marginal likelihood function of the variable is found and then maximized with respect to the population parameters. The algorithm used to optimize the likelihood function relies on the EM principle of Dempster, Laird, and Rubin (1977) and is known as the EM algorithm.[3] The advantage of the EM algorithm over other algorithms developed for numerical optimization is that it is robust to initial values.[4] Once the optimal values of the parameters are found, the parameters along with the data are used to draw the statistical inferences about the unobserved states. Recall that Sclove (1983) calculated what the likelihood function would have been if the regimes were observable, and then assumed that the actual historical states were those that would make the joint likelihood function of the changes in futures prices along with unobserved states as large as possible.

I use Hamilton's method to estimate two different model specifications. In the first specification, futures price changes are assumed to follow an autoregressive process as in

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[3] The EM algorithm is an iterative procedure composed of two steps, the expectation step (E-step) and the maximization step (M-step). Hence, its name.

[4] The EM algorithm also avoids problems that are associated with likelihood functions of switching regression models which are characterized by having many local maxima, singularities, and boundary problems.

Sclove (1983). In the second model, price changes are assumed to be independent as in Engel and Hamilton (1990).

Estimation of the Models

In this section I give the results from the estimation of the models hypothesized in the section above. Because I will consider out-of-sample prediction in the next section, the last 50 observations in every data set are not used for estimation.

The ARCH-M and GARCH-M Models

I started by fitting the ARCH-M(p) model to the five contracts. To fit a model, the value of p, the lag in the variance equation, is prespecified. The log likelihood function, given by

$$L(\phi) = T^{-1}\sum_{t=1}^{T} L_t(\phi) \tag{4.12}$$

$$L_t(\phi) = -\frac{1}{2}\log h_t - \frac{e_t^2}{2h_t}$$

where $\phi'=(\gamma,\alpha_0,\alpha_1,\ldots,\alpha_p)$ and the constant term omitted, is then maximized with respect to ϕ. Maximization of the likelihood function is carried out using the Berndt, Hall, Hall, Hausman (1974) numerical optimization technique. For large samples, such as the data representing a Treasury Bill futures contract, choice of the initial values of the parameters is not crucial.

Several specifications were tried using one to five lags in the variance equation and using the standard deviation and the variance in the mean equation. To choose the appropriate specification a standard likelihood ratio statistic was used. If $L(\phi_n)$ and $L(\phi_a)$ are the maximum likelihood function values under the null and alternative hypothesis respectively, then the statistic

$$-2\,[L(\phi_n) - L(\phi_a)] \sim \chi^2(k) \qquad \textbf{(4.13)}$$

where k, the degrees of freedom, is the difference in the number of parameters under the null and the alternative.

Table 4.1 gives the results of fitting the ARCH-M model to the data. The t-statistics are in parentheses. In addition to estimating the models, several diagnostic tests were conducted. First, a K-2 degree of freedom likelihood ratio test, where K is the number of parameters estimated in the model, for the null hypothesis that the endogenous variable follows a normal model with constant mean and variance is conducted. This test is applied as a necessary condition for applying the ARCH-M model to the data. Second, both the coefficients of skewness and kurtosis of the standardized residuals, i.e., $e_t/\surd h_t$, are given as an informal check of goodness of fit. If the model fits well, then the standardized residuals should satisfy the assumptions made before estimation. With respect to the their third and fourth central moments, this means that the coefficients of skewness

Table 4.1

ARCH-M(p) Model Estimates

Parameter	88(3)	88(6)	88(9)	88(12)	89(3)
δ	-0.048 (-5.23)	----	0.055 (4.44)	----	----
γ	0.641 (6.50)	0.037 (.822)	-0.637 (-4.65)	-0.015 (-.288)	-0.011 (-.199)
α_0	0.004 (7.93)	0.004 (8.76)	0.006 (10.90)	0.005 (9.24)	0.005 (10.81)
α_1	0.008 (0.320)	0.060 (1.89)	0.483 (11.90)	0.117 (2.32)	0.129 (2.52)
α_2	0.057 (1.86)	0.049 (2.03)	0.048 (1.11)	0.051 (2.10)	0.033 (1.95)
α_3	0.187 (3.43)	0.117 (2.14)	----	0.092 (1.65)	0.073 (1.73)
α_4	0.444 (7.12)	0.314 (8.89)	----	0.366 (6.74)	0.336 (6.09)
LR(K-2)	170.57	124.45	78.922	121.29	117.16
Skewness	-0.115	-0.486	0.233	0.185	0.341
Kurtosis	3.867	8.179	7.544	5.989	6.307
$L\text{-}B_1$	26.29	8.630	9.731	8.665	9.085
$L\text{-}B_2$	18.29	4.881	16.71	8.453	5.726

and kurtosis of the standardized residuals should be approximately equal to 0 and 3 respectively. Lastly, two Ljung-Box statistics for twelfth-order serial correlation, both $\chi^2(12)$, are conducted. The first tests the normalized residuals and the second tests the squared normalized residuals. They are denoted by $L\text{-}B_1$ and $L\text{-}B_2$ respectively.

The purpose of these last two tests is to see whether or not after modeling the residuals contain any linear dependence or any ARCH effects. If these tests fail to reject the null, then they are indicators that the ARCH-M models fit the data well.

In every data set, the likelihood ratio tests demonstrate that the ΔP_t does not have a constant mean and variance, that is, ΔP_t is better described by a model that allows for variation in the mean and variance of the process. Again, for every data set, the skewness coefficient is not very different from that found under a normal distribution, but the coefficient of kurtosis is somewhat high indicating that the standardized residuals have distributions that have very heavy tails. Although it is assumed that the standardized residuals should resemble standardized normal random variables, the ARCH-M model is not designed to model dependence found in moments greater than the second-order. Lastly, in every data set, the Ljung-Box statistics are insignificant at the 5% level of significance except for the test of linear dependence in contract 88(3).

A natural generalization to the ARCH-M model is the GARCH-M model. As in the ARCH-M model, several lag structures were tried for the variance equation and for every lag structure tried, the mean equation was estimated with and without a constant term. The GARCH-M is estimated the same way as the ARCH-M model except that $\phi' = (\gamma, \alpha_0, \alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q)$ where the β's are the coefficients on the lagged h_t's in the variance equation. Once again, to find the correct model speoification for the data the likelihood ratio test described above was used. For every data set, the GARCH-M(1,1) specification appeared to fit the data best. In addition, for every data set, a mean equation without a constant term seemed to be more appropriate than a mean equation with one. When compared to the ARCH-M model, a mean equation without a constant term may appear inconsistent. However, because the GARCH-M contains a lagged variance term in the variance equation, the conditional variance at time t may contain the information that the constant term proxied for in the ARCH-M specification. Hence, the constant term may not be necessary in the GARCH-M specification.

Table 4.2 gives the results of fitting the GARCH-M model to the data. The t-statistics are in parentheses. In each data set the likelihood ratio tests again indicate that the ΔP_t is described better by a model that allows for variation in the conditional mean and variance of the process over time. The coefficients of skewness are slightly higher than those reported under the ARCH-M models, but they still are not too far from what is acceptable for normal processes. Again, the coefficients of kurtosis are somewhat high. This may be for the same reason mentioned above. For every data set, the

Table 4.2

GARCH-M(p,q) Model Estimates

Parameter	88(3)	88(6)	88(9)	88(12)	89(3)
γ	0.017 (.343)	0.019 (.378)	-0.031 (-.610)	-0.043 (-.906)	-0.050 (-1.06)
α_0	0.0003 (2.40)	0.0004 (2.87)	0.0009 (2.83)	0.0013 (3.89)	0.0014 (3.67)
β_1	0.779 (26.1).	0.836 (40.5)	0.731 (14.1)	0.696 (12.3)	0.6903 (11.2)
α_1	0.221 (8.95)	0.128 (11.0)	0.195 (7.45)	0.167 (7.63)	0.139 (8.67)
LR(K-2)	148.97	106.44	94.973	102.37	95.66
Skewness	-0.172	-0.126	0.465	0.487	0.706
Kurtosis	6.129	10.86	8.510	8.411	9.590
L-B_1	9.15	6.86	7.980	7.955	8.438
L-B_2	21.43	6.27	8.765	6.914	6.174

Ljung-Box test statistics are insignificant at the 5% level of significance.

The Bilinear Model

Next, the data were modeled by a bilinear process. As Subba Rao (1977) pointed out, the problem of estimating the parameters of a bilinear model does not differ, in principle, from that of estimating the parameters of a linear model. Thus, if one assumes that e_t is a Guassian process, then given observations on the series for $t=1,\ldots,T$, the likelihood function may, for large T, be written approximately as, where θ denotes the set of parameters (a, b, d) and where, for

$$L(\theta) = \exp[\frac{-1}{2\sigma_e^2} \sum_{t=1}^{T} e_t^2] \tag{4.14}$$

each value of θ, e_t may be computed recursively from

$$e_t = P_t - aP_{t-1} - bP_{t-1}e_{t-1} - de_{t-1}. \tag{4.15}$$

The maximum likelihood estimate, $\hat{\theta}$, is therefore obtained by minimizing

$$V(\theta) = \sum_{t=1}^{T} e_t^2$$

with respect to each element of θ. Given a set of initial estimates, $\hat{\theta}_o$, a standard Newton-Raphson iterative technique may be used to find the value, $\hat{\theta}$, which minimizes $V(\theta)$.

Several initial values were chosen for the parameters; however, for the data sets considered in this paper, they did not significantly change the final estimates of a, b, and d. In addition, the model gave estimates for the parameter a that were very close to 1. Hence, it was considered parsimonious to instead estimate the model

$$(P_t - P_{t-1}) = bP_{t-1}e_{t-1} + de_{t-1} + e_t .$$

Thus, the estimates and in parentheses their corresponding t-statistics of the parameters b and d are given in table 4-3.

Table 4.3

Bilinear Model Estimates

Parameters	88(3)	88(6)	88(9)	88(12)	89(3)
b	-0.081 (-1.210)	-0.078 (-0.968)	-0.014 (-0.175)	0.038 (0.556)	-0.059 (-0.682)
d	7.573 (1.222)	7.273 (0.972)	1.318 (0.177)	-3.609 (-0.566)	5.398 (0.685)
skewness	1.457	0.561	0.960	1.406	1.275
kurtosis	16.991	19.642	18.038	20.149	19.799
W	1.719	1.509	1.146	1.139	1.376

In every data set, the parameter estimates are insignificant at the 5% level of significance. However, the model did appear useful in that a joint F-test for model significance was only marginally insignificant.

As for the residual diagnostics, W is a statistic formed by considering a second order covariance analysis on the squares of the residuals. It is suggested as a test for independence by Granger and Anderson (1978b), and is asymptotically distributed as normal with mean zero and variance unity. This test, as opposed to other tests for serial correlation, is used because standard tests tend to pick up the functional relationship of the residuals specified by the model. The results indicate that the test cannot reject the hypothesis that the residuals are independent. However, under the assumption that the residuals are Gaussian, the coefficients of skewness and kurtosis appear problematic.

Overall, it is quite clear that the bilinear model is dominated by the unit root process of the data.[5] However, in the next section the estimated models will also be judged by looking at their predictive ability.

A Time-Varying Parameter Model

Next, two time-varying parameter models were estimated. To fit both of these models the following procedure was used: Given the model specified in equation (4.7) let b_t be the estimate of β_t and S_t an estimate of its variance, i.e., $var(b_t)=S_t$. At time t, the observation P_t is known. An estimate of β_t, therefore, from the first equation

$$P_t = \beta_t P_{t-1} + \epsilon_t$$

is $(P_t P_{t-1})/P^2_{t-1}$ or simply P_t/P_{t-1} with variance σ^2/P^2_{t-1}. Also, from the second equation

$$\beta_t = \alpha\beta_{t-1} + u_t$$

an estimate for β_t is $a \cdot b_{t-1}$, where a is the estimate of α, and its variance is $(a^2 S_{t-1}+\theta^2)$. θ^2 is added to S_{t-1} because of the error term u_t. The time-varying parameter estimate of β_t is found by combining these two estimates with weights in inverse proportion to their variances,

[5] Other specifications were fit and in all cases the models were dominated by the unit root component of the data.

$$b_t = \left[\frac{1}{a^2 S_{t-1} + \theta^2} + \frac{P_{t-1}^2}{\sigma^2}\right]^{-1} \left[\frac{b_{t-1}}{a^2 S_{t-1} + \theta^2} + \frac{P_t P_{t-1}}{\sigma^2}\right] \tag{4.16}$$

where the variance S_t is given by

$$S_t^{\cdot} = \left[\frac{1}{a^2 S_{t-1} + \theta^2} + \frac{P_{t-1}^2}{\sigma^2}\right]^{-1} .$$

These recursive relations yield estimates that are equivalent to those given by the Kalman Filter. To begin the recursions initial estimates are needed. The initial estimates are

$$b_t = \frac{P_t P_{t-1}}{P_{t-1}^2} = \frac{P_t}{P_{t-1}}$$

$$\hat{\sigma}^2 = \frac{1}{T-1} \sum_2^T (P_t - b_t P_{t-1})^2$$

$$a = \frac{1}{T-1} \sum_2^T \frac{b_t b_{t-1}}{b_t^2} \tag{4.17}$$

$$\theta^2 = \frac{1}{T-1} \sum_2^T (b_t - a b_{t-1})^2$$

$$S_1 = \frac{\sigma^2}{P_1^2} + \theta^2$$

$$b_1 = \frac{P_1 P_2}{P_1^2} = \frac{P_2}{P_1} .$$

From these initial estimates the first set of iterations are conducted. Given b_1, and after a new series of b_t's are iterated, new estimates of $\hat{\sigma}^2$, $\hat{\theta}^2$, a, and S_1 are constructed. With these new estimates, a new series of b_t's are constructed. This procedure continues until the new estimates are not different from the old or the series $\{b_t\}$ does not change.

The first model fit was the one given above and the second constrained $\alpha=1$. In both cases, when the series $\{b_t\}$ converged, the values b_t all hovered around 1 with a variance at each time period roughly equal to 2.49943D-12. Given this result two different tests for stability were conducted. In both tests the null hypothesis is that the parameters of the model are time invariant. The first test has an alternative hypothesis of unstable regression coefficients and is conducted by splitting the entire sample into several arbitrary nonoverlapping subsamples and calculating the between-group-over-within-groups ratio of mean squares. Under the null hypothesis of stable coefficients, the test statistic is distributed $F(p-1, T-p)$ since there is only one regressor in the mean equation, $P_t = \beta_t P_{t-1} + \epsilon_t$, and p is the number of nonoverlapping subsamples. Test results for my data sets could not reject the null hypothesis of stable coefficients. The second test had the same null, but was against the alternative hypothesis of random-walk coefficients. This more specific test was conducted by considering the heteroskedastic form that ordinary least squares regression residuals have under an alternative hypothesis of random walk coefficients. Knowing that the form depends on $t \cdot P_{t-1}$, one can follow Breusch and Pagan (1979), by using one half times the explained sum of squares from a regression of $\hat{\epsilon}^2_t / \sigma^2$ on $t \cdot P^2_{t-1}$, where this statistic is $\chi^2(1)$. This statistic was also insignificant at the 5% level of significance in all data sets. Given the poor fit of the TVP model to futures data, I do not report the results nor use the model for predictive purposes.

A Time-Series Segmentation Model

The procedure used for estimating Sclove's (1983) model was described in the section above. From a first-order autoregressive representation to a third-order one, this model failed to conclude that the data were produced by two different regimes or states--in all cases, the algorithm converged to one state of the world. On the surface this result would not appear as bad if the parameters of the single autoregressive representation were stable. However, when estimating the model using several different combinations of initial estimates for the two autoregressive specifications the parameter estimates varied terribly. For each combination of initial estimates, the parameter estimates were different. For this reason, this model was abandoned and not used for predictive purposes.

A Stochastic, Segmented Trends Model

The concept behind estimation of the model is straightforward. Six population parameters determine the probability law for ΔP_t. These parameters are given by $\theta = (\mu_1, \mu_2, \sigma_1, \sigma_2, p_{11}, p_{22})$. The unconditional distribution of the state of the first observation, $p(s_1 = 1 | \theta) = \rho$, where

$$\rho = \frac{(1 - p22)}{(1 - p11) + (1 - p22)} \quad . \qquad \textbf{(4.18)}$$

The joint probability distribution for the sample size T and unobserved states s is given by

$$p(\Delta P_1, \ldots, \Delta P_T, s_1, \ldots, s_T; \theta) = p(\Delta P_T | s_T; \theta) * p(s_T | s_{T-1}; \theta)$$
$$* \; p(\Delta P_{T-1} | s_{T-1}; \theta) p(s_{T-1} | s_{T-2}; \theta) * \ldots * \qquad \textbf{(4.19)}$$
$$p(\Delta P_1 | s_1; \theta) p(s_1; \theta) .$$

The sample likelihood function is then just the summation of the joint probability distribution over all possibilities $(s_1, \ldots, s_T)$, i.e.,

$$\sum_{s_1=1}^{2} \cdots \sum_{s_T=1}^{2} p(\Delta P_1, \ldots \Delta P_T, s_1, \ldots, s_T; \theta) \quad . \qquad \textbf{(4.20)}$$

A simpler way to evaluate the sample likelihood function then the 2^T summations that it would ordinarily require is to use the algorithm provided by Hamilton (1989). Using this algorithm and incorporating a Bayesian prior, following Hamilton (1988), for the parameters of the two states, the parameter estimates are given by the following equations:

$$\hat{\mu}_j = \frac{\sum_{t=1}^{T} \Delta P_t * p(s_t=j \mid \Delta P_1, \ldots, \Delta P_T; \hat{\theta})}{\nu + \sum_{t=1}^{T} p(s_t=j \mid \Delta P_1, \ldots, \Delta P_T; \hat{\theta})} \tag{4.21}$$

$$\hat{\sigma}_j^2 = \left[\frac{1}{\alpha + (1/2)\sum_{t=1}^{T} p(s_t=j \mid \Delta P_1, \ldots, \Delta P_T: \hat{\theta})}\right] *$$

$$\left[\beta + (1/2)\sum_{t=1}^{T} (\Delta P_t - \hat{\mu}_j)^2 * p(s_t=j \mid \Delta P_1, \ldots, \Delta P_T; \hat{\theta}) + (1/2) * \nu * (\hat{\mu}_j)^2\right]$$

$$\hat{p}_{11} = \frac{\sum_{t=2}^{T} p(s_t=1, s_{t-1}=1 \mid \Delta P_1, \ldots, \Delta P_T; \hat{\theta})}{\sum_{t=2}^{T} p(s_{t-1}=1 \mid \Delta P_1, \ldots, \Delta P_T; \hat{\theta}) + \hat{\rho} - p(s_1=1 \mid \Delta P_1, \ldots, \Delta P_T; \hat{\theta})}$$

$$\hat{p}_{22} = \frac{\sum_{t=2}^{T} p(s_t=2, s_{t-1}=2 \mid \Delta P_1, \ldots, \Delta P_T; \hat{\theta})}{\sum_{t=2}^{T} p(s_{t-1}=2 \mid \Delta P_1, \ldots, \Delta P_T; \hat{\theta}) + \hat{\rho} - p(s_1=1 \mid \Delta P_1, \ldots, \Delta P_T; \hat{\theta})}.$$

The Bayesian prior, incorporated by using the parameters α, β, and ν, is used to avoid singularities of the likelihood function. Note that the maximum likelihood estimates are just a special case of the diffuse prior $\alpha=\beta=\nu=0$ and the use of priors that are nonzero shifts the maximum likelihood estimates in the direction of concluding that there is no difference between the two regimes.

Given this estimation procedure, table 4.4 provides the results of applying Engel and Hamilton's (1990) model to the futures price data. I encountered the same problem with Hamilton's model as that found with Sclove's model when specifying different autoregressive representations under the hypothesized two states of the world. Hence, it appears that it is not the estimation procedure, but the specification which does not appeal to the data. One reason for the failure of both models may be that they require stationary variables. Recall that figures 6-10 show that, for these contracts, changes in futures prices occur in the opposite direction very frequently. This would lead one to believe that a negative first-order autoregressive representation would dominate a positive one. Also, the levels shown in figures 1-5, exhibit

Table 4.4

Stochastic, Segmented Trends Model Estimates

	88(3)	88(6)	88(9)	88(12)	89(3)
μ_1	.07159 (.10876)	.03715 (.18825)	.15040 (.23603)	.12420 (.22863)	.10663 (.23041)
μ_2	-.00181 (.00501)	-.00117 (.00544)	-.00482 (.00579)	-.00414 (.00503)	-.00329 (.00499)
p_{11}	.84686 (.10349)	.82381 (.16311)	.80862 (.17772)	.81461 (.16656)	.79933 (.18167)
p_{22}	.99395 (.00437)	.99350 (.00479)	.99645 (.00362)	.99689 (.00318)	.99664 (.00351)
σ_1^2	.18373 (.07435)	.32440 (.17664)	.34941 (.20679)	.34589 (.20153)	.35304 (.20617)
σ_2^2	.00997 (.00075)	.01086 (.00084)	.01139 (.00088)	.01041 (.00074)	.00999 (.00073)
Ps_N	.0041	.0017	.0008	.0491	.0007
ρ	.0380	.0356	.0182	.0165	.0165

Ps_N is the conditional probability $P(s_N=1|\Delta P_N,...,P_1)$ where N=T-50 and T is the number of observations in a given contract. The standard errors are in parentheses.

sporadic movements even though the series is generally moving upward or downward or appears to have a 'long swing'. These sporadic movements are quite different than what Hamilton encountered when modeling exchange rates. Engel and Hamilton's (1990) model was able to fit the data somewhat better. By not specifying an autoregressive representation, this form of the model was able to distinguish between two different states. However, even though Hamilton's (1990) model was able to fit the futures data better than his other models, ninety-five percent of all observations in all of my data sets were distinguished as coming from the second state of the world. Hence, this model still does not seem fully appropriate for financial futures price data.

By looking at table 4.4, the means of the distributions μ_1 and μ_2 are not significant at conventional sizes in any of the data sets. This indicates that the trend that the data follows at any given time period is not well specified. Also, in every contract, the variance for the first distribution is not significant. Lastly, the conditional probability that the last observation's state is 1 and the probability that the first state is 1 are both very small. This yields further evidence that the model may not be appropriate for the data.

Prediction and Comparison of the Estimated Models

For each of the estimated models, prediction is carried out for a 5-day horizon up through a 50-day horizon. In total, 10 predictions will be made for each model. The mean square error (MSE) of prediction and the Theil U statistic for each horizon is first compared to the MSE of prediction and the U statistic of the simple random walk model. Then, the MSE of prediction and the U statistic of each of the estimated models is compared against one another. The criterion for choosing the best model is simple: The lower the MSE of prediction and the lower the U statistic, the better the prediction, and, the better the prediction, the better the model.

The MSE of prediction is formally defined as:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left[\frac{Pr_t - A_t}{A_{t-1}}\right]^2 \tag{4.22}$$

where Pr_t is the predicted value at time t, A_t is the actual value at time t, and n is forecast horizon. It is the simplest measure of forecast accuracy and it is the basis for all other measures. The Theil U statistic, a function of the MSE is given by:

$$U_{\Delta} = \sqrt{\frac{\frac{1}{n}\sum_{t=1}^{n}(\Delta Pr_t - \Delta A_t)^2}{\frac{1}{n}\sum_{t=1}^{n}(\Delta A_t)^2}} \tag{4.23}$$

where ΔPr_t is the first difference of the predicted values, ΔA_t is the first differences of the actual values, and n is the forecast horizon.

The MSE is an overall measure of forecast performance that is based purely on the forecast errors. On the other hand, the Theil U statistic given in terms of differences, is used to measure the model's ability to track turning points in the data. By using both of these measures the best model should be detected.

Ten tables are given below, one for each forecast horizon. The first horizon represents the first five days immediately following the estimation period, the second represents the first ten days immediately following the estimation period, the third the first fifteen days, and so forth. The longest forecast horizon analyzed is fifty trading days.

Table 4.5

Prediction Under the First Forecast Horizon

	MSE x 10^6	U-STATISTIC
RAN.WALK$_1$	2.470	2.698
RAN.WALK$_2$	0.340	1.107
RAN.WALK$_3$	3.590	1.065
RAN.WALK$_4$	0.154	1.462
RAN.WALK$_5$	2.443	1.397
ARCH-M$_1$	2.971	2.769
ARCH-M$_2$	0.335	1.110
ARCH-M$_3$	4.409	1.066
ARCH-M$_4$	0.159	1.485
ARCH-M$_5$	2.443	1.398
GARCH-M$_1$	2.454	2.704
GARCH-M$_2$	0.333	1.108
GARCH-M$_3$	3.556	1.065
GARCH-M$_4$	0.164	1.462
GARCH-M$_5$	2.423	1.400
BILINEAR$_1$	2.442	2.659
BILINEAR$_2$	0.368	1.095
BILINEAR$_3$	3.581	1.068
BILINEAR$_4$	0.168	1.515
BILINEAR$_5$	2.441	1.406
SEG.TRENDS$_1$	2.563	2.686
SEG.TRENDS$_2$	0.360	1.099
SEG.TRENDS$_3$	3.248	1.055
SEG.TRENDS$_4$	0.173	1.462
SEG.TRENDS$_5$	2.328	1.397

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

Table 4.6

Prediction Under the Second Forecast Horizon

	MSE x 10^6	U-STATISTIC
RAN.WALK$_1$	2.570	1.770
RAN.WALK$_2$	0.193	1.327
RAN.WALK$_3$	2.179	1.593
RAN.WALK$_4$	0.182	1.303
RAN.WALK$_5$	1.767	1.427
ARCH-M$_1$	2.627	1.693
ARCH-M$_2$	0.191	1.330
ARCH-M$_3$	2.622	1.704
ARCH-M$_4$	0.184	1.312
ARCH-M$_5$	1.768	1.428
GARCH-M$_1$	2.581	1.771
GARCH-M$_2$	0.190	1.327
GARCH-M$_3$	2.183	1.597
GARCH-M$_4$	0.189	1.301
GARCH-M$_5$	1.751	1.428
BILINEAR$_1$	2.559	1.764
BILINEAR$_2$	0.203	1.281
BILINEAR$_3$	2.172	1.589
BILINEAR$_4$	0.192	1.322
BILINEAR$_5$	1.763	1.431
SEG.TRENDS$_1$	2.465	1.767
SEG.TRENDS$_2$	0.218	1.324
SEG.TRENDS$_3$	2.337	1.595
SEG.TRENDS$_4$	0.204	1.296
SEG.TRENDS$_5$	1.686	1.426

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

Table 4.7

Prediction Under the Third Forecast Horizon

	MSE x 10^6	U-STATISTIC
$RAN.WALK_1$	2.366	1.631
$RAN.WALK_2$	0.313	1.301
$RAN.WALK_3$	1.553	1.596
$RAN.WALK_4$	0.181	1.205
$RAN.WALK_5$	1.260	1.391
$ARCH\text{-}M_1$	2.586	1.551
$ARCH\text{-}M_2$	0.315	1.304
$ARCH\text{-}M_3$	1.998	1.703
$ARCH\text{-}M_4$	0.182	1.211
$ARCH\text{-}M_5$	1.260	1.393
$GARCH\text{-}M_1$	2.362	1.631
$GARCH\text{-}M_2$	0.314	1.300
$GARCH\text{-}M_3$	1.552	1.600
$GARCH\text{-}M_4$	0.183	1.204
$GARCH\text{-}M_5$	1.244	1.392
$BILINEAR_1$	2.360	1.628
$BILINEAR_2$	0.323	1.265
$BILINEAR_3$	1.546	1.592
$BILINEAR_4$	0.188	1.223
$BILINEAR_5$	1.253	1.395
$SEG.TRENDS_1$	2.559	1.629
$SEG.TRENDS_2$	0.285	1.298
$SEG.TRENDS_3$	1.797	1.597
$SEG.TRENDS_4$	0.195	1.199
$SEG.TRENDS_5$	1.257	1.391

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

Table 4.8

Prediction Under the Fourth Forecast Horizon

	MSE x 10^6	U-STATISTIC
RAN.WALK$_1$	1.856	1.673
RAN.WALK$_2$	0.349	1.470
RAN.WALK$_3$	1.443	1.486
RAN.WALK$_4$	0.256	1.105
RAN.WALK$_5$	1.072	1.345
ARCH-M$_1$	2.079	1.657
ARCH-M$_2$	0.349	1.475
ARCH-M$_3$	2.005	1.579
ARCH-M$_4$	0.258	1.109
ARCH-M$_5$	1.072	1.346
GARCH-M$_1$	1.851	1.673
GARCH-M$_2$	0.348	1.469
GARCH-M$_3$	1.436	1.489
GARCH-M$_4$	0.259	1.104
GARCH-M$_5$	1.062	1.346
BILINEAR$_1$	1.851	1.668
BILINEAR$_2$	0.346	1.409
BILINEAR$_3$	1.436	1.482
BILINEAR$_4$	0.269	1.119
BILINEAR$_5$	1.066	1.348
SEG.TRENDS$_1$	2.103	1.670
SEG.TRENDS$_2$	0.405	1.471
SEG.TRENDS$_3$	1.563	1.484
SEG.TRENDS$_4$	0.381	1.108
SEG.TRENDS$_5$	1.209	1.343

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

Table 4.9

Prediction Under the Fifth Forecast Horizon

	MSE x 10^6	U-STATISTIC
RAN.WALK$_1$	1.654	1.638
RAN.WALK$_2$	0.389	1.508
RAN.WALK$_3$	1.171	1.558
RAN.WALK$_4$	0.484	1.339
RAN.WALK$_5$	0.956	1.345
ARCH-M$_1$	1.802	1.598
ARCH-M$_2$	0.390	1.510
ARCH-M$_3$	1.688	1.661
ARCH-M$_4$	0.485	1.339
ARCH-M$_5$	0.955	1.346
GARCH-M$_1$	1.651	1.638
GARCH-M$_2$	0.391	1.508
GARCH-M$_3$	1.164	1.563
GARCH-M$_4$	0.491	1.335
GARCH-M$_5$	0.941	1.346
BILINEAR$_1$	1.649	1.632
BILINEAR$_2$	0.385	1.439
BILINEAR$_3$	1.164	1.554
BILINEAR$_4$	0.495	1.325
BILINEAR$_5$	0.947	1.350
SEG.TRENDS$_1$	1.898	1.636
SEG.TRENDS$_2$	0.392	1.508
SEG.TRENDS$_3$	1.583	1.560
SEG.TRENDS$_4$	0.843	1.340
SEG.TRENDS$_5$	1.094	1.345

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

Table 4.10

Prediction Under the Sixth Forecast Horizon

	MSE x 10^6	U-STATISTIC
RAN.WALK$_1$	1.555	1.666
RAN.WALK$_2$	0.388	1.484
RAN.WALK$_3$	1.057	1.560
RAN.WALK$_4$	0.423	1.348
RAN.WALK$_5$	0.811	1.342
ARCH-M$_1$	1.711	1.639
ARCH-M$_2$	0.388	1.485
ARCH-M$_3$	1.530	1.656
ARCH-M$_4$	0.424	1.348
ARCH-M$_5$	0.810	1.343
GARCH-M$_1$	1.553	1.666
GARCH-M$_2$	0.389	1.484
GARCH-M$_3$	1.053	1.564
GARCH-M$_4$	0.427	1.344
GARCH-M$_5$	0.798	1.343
BILINEAR$_1$	1.548	1.658
BILINEAR$_2$	0.384	1.423
BILINEAR$_3$	1.052	1.557
BILINEAR$_4$	0.431	1.335
BILINEAR$_5$	0.803	1.346
SEG.TRENDS$_1$	1.844	1.664
SEG.TRENDS$_2$	0.453	1.485
SEG.TRENDS$_3$	1.847	1.563
SEG.TRENDS$_4$	0.867	1.342
SEG.TRENDS$_5$	1.088	1.397

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

Table 4.11

Prediction Under the Seventh Forecast Horizon

	MSE x 10^6	U-STATISTIC
RAN.WALK$_1$	1.410	1.665
RAN.WALK$_2$	0.469	1.481
RAN.WALK$_3$	4.191	1.763
RAN.WALK$_4$	0.399	1.320
RAN.WALK$_5$	0.743	1.341
ARCH-M$_1$	1.533	1.625
ARCH-M$_2$	0.469	1.480
ARCH-M$_3$	5.433	1.883
ARCH-M$_4$	0.399	1.320
ARCH-M$_5$	0.743	1.342
GARCH-M$_1$	1.407	1.665
GARCH-M$_2$	0.471	1.481
GARCH-M$_3$	4.211	1.757
GARCH-M$_4$	0.399	1.317
GARCH-M$_5$	0.735	1.342
BILINEAR$_1$	1.402	1.656
BILINEAR$_2$	0.464	1.422
BILINEAR$_3$	4.205	1.755
BILINEAR$_4$	0.404	1.307
BILINEAR$_5$	0.737	1.346
SEG.TRENDS$_1$	1.779	1.663
SEG.TRENDS$_2$	0.501	1.481
SEG.TRENDS$_3$	5.215	1.763
SEG.TRENDS$_4$	0.878	1.319
SEG.TRENDS$_5$	1.303	1.342

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

Table 4.12

Prediction Under the Eighth Forecast Horizon

	MSE x 10^6	U-STATISTIC
RAN.WALK$_1$	1.279	1.653
RAN.WALK$_2$	0.569	1.426
RAN.WALK$_3$	3.757	1.752
RAN.WALK$_4$	0.390	1.310
RAN.WALK$_5$	0.711	1.345
ARCH-M$_1$	1.429	1.622
ARCH-M$_2$	0.569	1.426
ARCH-M$_3$	4.897	1.871
ARCH-M$_4$	0.390	1.309
ARCH-M$_5$	0.711	1.346
GARCH-M$_1$	1.276	1.653
GARCH-M$_2$	0.571	1.426
GARCH-M$_3$	3.776	1.757
GARCH-M$_4$	0.394	1.307
GARCH-M$_5$	0.703	1.346
BILINEAR$_1$	1.274	1.644
BILINEAR$_2$	0.561	1.379
BILINEAR$_3$	3.770	1.755
BILINEAR$_4$	0.399	1.298
BILINEAR$_5$	0.706	1.350
SEG.TRENDS$_1$	1.771	1.651
SEG.TRENDS$_2$	0.641	1.426
SEG.TRENDS$_3$	5.145	1.752
SEG.TRENDS$_4$	1.231	1.311
SEG.TRENDS$_5$	1.404	1.344

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

Table 4.13

Prediction Under the Ninth Forecast Horizon

	MSE x 10^6	U-STATISTIC
RAN.WALK$_1$	1.247	1.625
RAN.WALK$_2$	0.512	1.470
RAN.WALK$_3$	3.352	1.755
RAN.WALK$_4$	0.439	1.256
RAN.WALK$_5$	0.665	1.333
ARCH-M$_1$	1.418	1.600
ARCH-M$_2$	0.513	1.469
ARCH-M$_3$	4.419	1.874
ARCH-M$_4$	0.439	1.255
ARCH-M$_5$	0.665	1.334
GARCH-M$_1$	1.242	1.625
GARCH-M$_2$	0.571	1.760
GARCH-M$_3$	3.366	1.065
GARCH-M$_4$	0.438	1.254
GARCH-M$_5$	0.654	1.334
BILINEAR$_1$	1.243	1.614
BILINEAR$_2$	0.507	1.415
BILINEAR$_3$	3.362	1.758
BILINEAR$_4$	0.447	1.242
BILINEAR$_5$	0.658	1.338
SEG.TRENDS$_1$	1.933	1.624
SEG.TRENDS$_2$	0.600	1.470
SEG.TRENDS$_3$	5.091	1.756
SEG.TRENDS$_4$	1.280	1.256
SEG.TRENDS$_5$	1.454	1.333

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

Table 4.14

Prediction Under the Tenth Forecast Horizon

	MSE x 10^6	U-STATISTIC
RAN.WALK$_1$	1.164	1.626
RAN.WALK$_2$	0.470	1.470
RAN.WALK$_3$	3.018	1.755
RAN.WALK$_4$	0.448	1.252
RAN.WALK$_5$	0.638	1.326
ARCH-M$_1$	1.346	1.600
ARCH-M$_2$	0.471	1.469
ARCH-M$_3$	4.011	1.874
ARCH-M$_4$	0.447	1.251
ARCH-M$_5$	0.637	1.327
GARCH-M$_1$	1.159	1.626
GARCH-M$_2$	0.472	1.469
GARCH-M$_3$	3.031	1.759
GARCH-M$_4$	0.444	1.251
GARCH-M$_5$	0.625	1.327
BILINEAR$_1$	1.160	1.612
BILINEAR$_2$	0.466	1.414
BILINEAR$_3$	3.027	1.758
BILINEAR$_4$	0.456	1.239
BILINEAR$_5$	0.630	1.332
SEG.TRENDS$_1$	1.957	1.624
SEG.TRENDS$_2$	0.580	1.470
SEG.TRENDS$_3$	5.271	1.755
SEG.TRENDS$_4$	1.407	1.254
SEG.TRENDS$_5$	1.544	1.326

Note that the subscripts 1,...,5 on each of the models indicate the model for contracts 88(3), 88(6),...,89(3) respectively.

When compared to the random walk, the ARCH-M model predicts worse in short horizons. The MSE of prediction of the random walk model for all contracts is at least as small as that of the ARCH-M model in the first five horizons. The random walk also predicts better for horizons six through ten in three of the five data sets. By looking at the U-statistics it is also clear that the random walk is better able to predict turning points in the data. This is somewhat remarkable because the random walk by definition cannot predict turning points. Only in contract 88(3) does the ARCH-M predict turning points more accurately. Given these results the ARCH-M model does not seem appropriate for capturing the important nonlinearities of the data. Thus, even though there are theoretical reasons to believe that conditional moments are important determinants of asset prices[6], these results should question the extent to which the ARCH model has been used in modeling financial data, especially futures data.

The GARCH-M model performed somewhat better than the ARCH-M model, but still not sufficiently well to believe that the nonlinearities in the data were completely modeled. In two of the five data sets, the GARCH-M model had a smaller MSE of prediction than the random walk. When considering all five contracts, in twenty-nine of the possible fifty horizons the GARCH-M model had a smaller MSE of prediction. In addition, the prediction horizon did not appear to be important. Depending upon the particular contract, either the random walk or the GARCH-M model dominated in every horizon. However, the ability of the GARCH-M model to track turning points was much worse than that of the random walk. In three of the five data sets the random walk had smaller U-statistics. These results

[6] Many intertemporal asset-pricing models give rise to Euler equations that involve conditional expectations of marginal utilities across time periods, (see Lucas (1978)).

should not appear surprising since the ARCH-M model also could not outperform the simple linear model.

The bilinear model appeared to be the best model from all of the nonlinear models considered. The random walk, when compared to the bilinear model, had lower MSE of predictions in only one of the five contracts. The bilinear model was also better able to detect turning points. In thirty-one of the fifty possible comparisons, the bilinear model had lower U-statistics than the random walk. Like the comparison made between the GARCH-M model and the random walk, the prediction horizon did not appear to be important here either. The bilinear model dominated the random walk's performance for every horizon and in every contract except 88(12).

When compared to the random walk, the worst model was the stochastic, segmented trends (SST) model. The random walk performed better in every contract. The SST model had lower MSE's of prediction in six of the fifty possible cases. Given the poor results in the estimation period, these prediction results should not be surprising. Regardless of which futures contract was modeled, roughly ninety percent of the time the SST model estimated the data to come from state two.

The ability of the SST model to predict turning points was somewhat better than its overall performance. For contract 88(3), it was able to predict turning points better than the random walk in every horizon. Overall, the SST model had lower U-statistics in the earlier horizons.

Among just the nonlinear models, the SST model still has the largest MSE of prediction. However, if the performance of the models are just measured by looking at the MSE in the first horizon, then the SST model performs better than both the ARCH-M and bilinear models and better than the GARCH-M model in two of the five data sets.

The ARCH-M model performs worse than both the bilinear and GARCH-M models, although, for contract 88(12), the ARCH-M model has the lowest MSE of prediction out of all the models. However, its MSE of prediciton is still not much lower even for this contract.

The GARCH-M and bilinear models have nearly equal MSE of predictions. However, there are still some marked differences between their performances. GARCH-M outperforms the bilinear model in earlier horizons and the bilinear model is clearly the better model for horizons 6 through 10.

Considering just the ability to predict turning points, the bilinear model is undoubtedly the best model. However, the SST model had the lowest U-statistics when considering just the first horizon. GARCH-M is better than the ARCH-M model given this criterion, and the ARCH-M model is definitely the worst of all models, including the SST model, for all horizons.

Given these two prediction criteria, the bilinear model is able to model futures prices better than every nonlinear model considered. Against the random walk, it also performed the best. It seems straightforward to conclude then that bilinear models are the best for T-bill futures prices, and that the popular family of ARCH models may be somewhat abused when used to model financial data.

CHAPTER 5
SUMMARY AND CONCLUSION

Several important issues relating to the time series properties of futures prices have been studied in the previous chapters. These include the stationarity of 90-day U.S. Treasury Bill futures prices, the validity of the random walk hypothesis, the existence of nonlinear dependence, and the exploitability of the nonlinear dependence that is found in futures prices in terms of prediction. Results in chapter 2 clearly confirm that Treasury Bill futures prices are nonstationary. In addition, I find that the best random walk model is one with neither a drift nor trend term. These results confirm the results of other studies of financial futures prices and provide a necessary condition for the random walk hypothesis to be affirmed.

It has been shown by using both nonparametric and parametric tests of dependence that first differences of futures prices contain no significant linear dependence. On the other hand, the Brock, Diechert, and Scheinkman test, the ARCH specification test, and Tsay's test for nonlinearity, all clearly demonstrated that nonlinear dependence is present. Thus, the random walk hypothesis cannot be verified. Hseih's test to distinguish between additive and multiplicative nonlinear dependence indicates that the data primarily contain multiplicative dependence. However, using two predictive criteria in chapter 4, I find that the bilinear model is the best nonlinear model for the data. Given this result and the fact that a bilinear process exhibits additive nonlinear dependence, it is not clear that Hseih's test is powerful enough to detect the different types of nonlinear dependence found in the data.

As mentioned above, the forecasting performance of the bilinear model is better than that of the random walk model and the other nonlinear models considered. Time-varying parameter models and several versions of Sclove's (1983) time series segmentation model were found to be inappropriate for these futures prices. The GARCH-M model was the second best, the ARCH-M model third best, and Hamilton's (1989) stochastic, segmented trends model worst of all the nonlinear models estimated. However, in earlier horizons, Hamilton's model was able to predict turning points better than most models. The most important results of chapter 4 are that: nonlinear dependence can be exploited for predictive purposes; the bilinear model is the best model for these futures data; some nonlinear models predict better than the random walk; and that the ARCH family of models are probably being misused or abused.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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